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RESEARCH ARTICLE

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Option-implied betas and the cross section of stock returns

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We investigate the cross-sectional relationship between stock returns and a number of measures of option-implied beta. Using portfolio analysis, we show that the method proposed by Buss and Vilkov (2012, *The Review of Financial Studies*, 2525, 3113–3140) leads to a stronger relationship between implied beta and stock returns than other approaches. However, using the Fama and MacBeth (1973, *Journal of Political Economy*, 8181, 607–636) cross-section regression methodology, we show that the relationship is not robust to the inclusion of other firm characteristics. We further show that a similar result holds for implied downside beta. We, therefore, conclude that there is no robust relation between option-implied beta and returns.

KEYWORDS
cross section, downside beta, option-implied beta, stock returns

JEL CLASSIFICATION
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1 | INTRODUCTION

The capital asset pricing model (CAPM), developed independently by Sharpe (1964), Lintner (1965), and Mossin (1966), predicts that the expected return of a stock should be a positive linear function of its market beta, and unrelated to all other characteristics of the stock. These predictions of the CAPM have been empirically tested in many studies.[1] However, these studies typically estimate the unobserved beta using historical data on stock returns. As noted by McNulty, Yeh, Schulze, and Lubatkin (2002), the use of historical stock returns to estimate market beta is problematic, since it leads to sensitivity to minor changes in the sample period used.

In an attempt to reduce the estimation error that arises from the use of historical data, a number of studies have developed estimators of market beta that exploit information about the covariance matrix of stock returns that is contained in option prices. French, Groth, and Kolari (1983; FGK) introduce a hybrid method to estimate market beta that combines an estimate of the correlation between the stock return and the market return from historical data with the ratio of stock-to-market implied volatility. Chang, Christoffersen, Jacobs, and Vainberg (2011; CCJV) use both option-implied skewness and volatility to estimate market beta. They find that the CCJV beta performs relatively well and can explain a sizeable proportion of cross-sectional variation in expected returns. Buss and Vilkov (2012; BV) compute option-implied beta using option-implied correlation and volatility. They find that in support of the CAPM, there is a monotonically increasing relation between BV beta and returns.

Buss and Vilkov (2012) compare their approach with both historical beta, and other option-implied betas, using tests based on portfolio sorting, and conclude that the BV beta performs best. In this paper, we investigate the robustness of these findings with respect to the inclusion of other firm-specific characteristics. We employ options on the S&P 500 index and its

[1]See, for example, Fama and French (1992), who find that the relation between market betas and average returns disappears during the more recent 1963–1990 period of U.S. stock return data even when beta is the only explanatory variable.

constituents to construct option-implied betas. Using both the portfolio sorting approach and the Fama and MacBeth (1973) regression approach, we compare the performance of four methods of estimating market beta: The historical beta, the FGK beta of French et al. (1983), the CCJV beta of Chang et al. (2011), and the BV beta of Buss and Vilkov (2012). We also develop several measures of option-implied downside beta based on existing implied beta methods combined with the downside correlation of Ang, Chen, and Xing (2002) and test the relationship between returns and downside beta using both the portfolio sorting approach and the Fama and MacBeth (1973) regression approach.

Our main findings are summarized as follows. First, we compare the historical, FGK, CCJV, and BV beta methods and find that the BV beta measure works best using portfolio sorts. A portfolio trading strategy that sells the stocks ranked in the bottom quintile by the BV beta and buys the stocks in the top quintile by the BV beta earns positive profit. This is consistent with the findings of Buss and Vilkov (2012). However, using the Fama and MacBeth (1973) regression approach, we find that the relationship between option-implied betas and stock returns is not significant, nor is it robust to other firm characteristics. Second, we develop measures of implied downside beta corresponding to the FGK and BV betas. Using portfolio sorts, we show that the BV downside beta has a positive relation with stock returns and performs better than both the historical downside beta and the FGK downside beta. It also offers an improvement over the standard BV beta. However, we again show that using the Fama and MacBeth (1973) regression approach, the relationship between implied downside beta and stock returns is not significant, nor is it robust to other firm-level characteristics. We, therefore, conclude that there is no robust, statistically significant relation between option-implied betas and stock returns.

Our paper contributes to the literature that examines the relation between market betas and stock returns. First, our study complements and extends the paper of Buss and Vilkov (2012), who compare these four beta methods using a portfolio analysis based on options on the S&P 500 index and its constituents from January 1996 to December 2009. Second, we contribute to the literature on downside beta. Post and Van Vliet (2004), Ang, Chen, and Xing (2006), and Tahir, Abbas, Sargana, Ayub, and Saeed (2013) report that the downside risk based CAPM outperforms the standard, variance-based CAPM.

The remainder of this paper is organized as follows: Section 2 describes the calculation of option-implied betas and downside betas. Section 3 describes the data and presents the summary statistics. Section 4 reports the empirical results for the relation between returns and implied beta, while Section 5 reports the corresponding results for implied downside beta. Section 6 concludes.

2 | OPTION-IMPLIED BETAS AND DOWNSIDE BETAS

2.1 | Historical beta

Let P denote the probability distribution function under the physical measure. The historical beta is calculated as

$$\beta_{i,M}^{\text{His}} = \rho_{i,M}\frac{\sigma_{i,t}^{P}}{\sigma_{M,t}^{P}}, \tag{1}$$

where $\sigma_{i,t}^{P}$ and $\sigma_{M,t}^{P}$ are the standard deviation of the returns of stock i and the index, respectively, and $\rho_{i,M}$ is the correlation between stock and index returns. Traditionally, the historical beta is calculated using the historical rolling-window method.

2.2 | FGK beta

French et al. (1983; FGK) introduce a hybrid estimation method using option-implied volatility to improve the performance of beta forecasts. Let Q denote the probability function under the risk-neutral measure. The FGK implied beta is defined as

$$\beta_{i,M}^{\text{FGK}} = \rho_{i,M}\frac{\sigma_{i,t}^{Q}}{\sigma_{M,t}^{Q}}, \tag{2}$$

where $\sigma_{i,t}^{Q}$ and $\sigma_{M,t}^{Q}$ are the option-implied volatility for stock i and the index, respectively, and $\rho_{i,M}$ is the correlation between historical stock and index returns.

2.3 | CCJV beta

Chang et al. (2011; CCJV) propose a one-factor model and assume zero skewness of the market return residual to propose a new market beta method by using both option-implied volatility and skewness. The CCJV implied beta is defined as

$$\beta_{i,M}^{\mathrm{CCJV}} = \left(\frac{SKEW_{i,t}^{Q}}{SKEW_{M,t}^{Q}}\right)^{\frac{1}{3}} \frac{\sigma_{i,t}^{Q}}{\sigma_{M,t}^{Q}}, \tag{3}$$

where $\sigma_{i,t}^{Q}$ and $\sigma_{M,t}^{Q}$ are option-implied volatility for stock i and the index, respectively. $SKEW_{i,t}^{Q}$ and $SKEW_{i,t}^{Q}$ are option-implied skewness on stock i and the index, respectively. $(SKEW_{i,t}^{Q}/SKEW_{M,t}^{Q})^{1/3}$ serves as a proxy for the risk-neutral correlation.

2.4 | BV beta

Buss and Vilkov (2012; BV) propose a measure of option-implied beta by combining option-implied correlation with option-implied volatility. First, we have one identifying restriction: The observed implied variance of the market index $(\sigma_{M,t}^{Q})^{2}$ equals the implied variance of a portfolio of all market index constituents $i = 1, \ldots, N$:

$$\left(\sigma_{M,t}^{Q}\right)^{2} = \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_i \omega_j \sigma_{i,t}^{Q} \sigma_{j,t}^{Q} \rho_{ij,t}^{Q}, \tag{4}$$

where $\sigma_{i,t}^{Q}$ denotes the implied volatility of stock i and ω_i represents the weights of stock i in the index.

Empirically, we use stock returns in the market index constituents to identify $N \times (N-1)/2$ physical correlations $\rho_{ij,t}^{P}$ and then transform these into implied correlations $\rho_{ij,t}^{Q}$[2].

$$\rho_{ij,t}^{Q} = \rho_{ij,t}^{P} - \alpha_t\left(1\text{-}\rho_{ij,t}^{P}\right), \tag{5}$$

where $\rho_{ij,t}^{P}$ is the expected correlation under the physical measure, and α_t denotes the parameter to be identified. Substituting the implied correlation in Equation 5 into restriction (4), we obtain the following formula to compute α_t:

$$\alpha_t = -\frac{\left(\sigma_{M,t}^{Q}\right)^{2} - \sum_{i=1}^{N} \sum_{j=1}^{N} \omega_i \omega_j \sigma_{i,t}^{Q} \sigma_{j,t}^{Q} \rho_{ij,t}^{Q}}{\sum_{i=1}^{N} \sum_{j=1}^{N} \omega_i \omega_j \sigma_{i,t}^{Q} \sigma_{j,t}^{Q} \left(1\text{-}\rho_{ij,t}^{P}\right)}. \tag{6}$$

After estimating implied volatility and correlation, we compute the BV beta as:

$$\beta_{iM,t}^{\mathrm{BV}} = \frac{\sum_{iM,t}^{N} \omega_j \sigma_{j,t}^{Q} \rho_{ij,t}^{Q}}{(\sigma_{M,t}^{Q})^{2}}. \tag{7}$$

[2]Buss and Vilkov (2012) note that the transformation must satisfy two technical conditions and two empirical observations. The two technical conditions are: (i) All correlations $\rho_{ij,t}^{Q}$ do not exceed one, and (ii) the correlation matrix is positive definite. Furthermore, the implied correlations are consistent with two empirical observations: (i) The implied correlation $\rho_{ij,t}^{Q}$ is higher than the correlation under the physical measure $\rho_{ij,t}^{P}$, (ii) the correlation risk premium is larger in magnitude for pairs of stocks that provide higher diversification benefits (i.e., low or negatively correlated stocks), and hence are exposed to a higher risk of losing diversification in bad times characterized by increasing correlations. The second empirical observation is supported by the negative correlation between the correlation under the objective measure and the correlation risk premium in Mueller, Stathopoulos, and Vedolin (2017).

2.5 | Implied downside betas

We use three methods to estimate downside beta, namely, the historical approach, and approaches based on the FGK and BV implied betas.[3] For the historical downside beta, we follow the semivariance beta approach of Hogan and Warren (1972). The computation of historical downside betas is as follows:

$$\beta_{\theta}^{D\text{-His}} = \frac{E[r_i r_M | r_M < \theta]}{E[r_M^2 | r_M < \theta]}. \tag{8}$$

where the numerator is the second lower partial comoment between the excess return of stock i, r_i, and the excess market return, r_M, and measures the comovement between the stock and the market during market downturns. The threshold, θ is used to define the downside market. In this paper, we set θ to be the mean of the excess market return, r_M.

The principle for modeling implied downside beta is based on modeling downside correlations. Ang et al. (2002) decompose downside beta into a conditional correlation term and a ratio of conditional total volatility to conditional market volatility. The downside correlation is given by:

$$\rho_{\theta}^{-} = \text{corr}\{r_i, r_M\} = \frac{E[r_i r_M | r_M < \theta]}{\sqrt{E[r_i^2 | r_M < \theta] E[r_M^2 | r_M < \theta]}}. \tag{9}$$

Following Ang et al. (2002), we combine downside correlation and option-implied volatility to obtain implied downside beta. We substitute the historical correlation of the FGK beta in Equation 2 by the downside correlation in Equation 9 to obtain the FGK implied downside beta:

$$\beta_{\theta}^{D\text{-FGK}} = \rho_{\theta}^{-} \frac{\sigma_{i,t}^{Q}}{\sigma_{M,t}^{Q}}. \tag{10}$$

For the BV beta method, we use individual stock returns satisfying $[r_i | r_M < \theta]$ to calculate the physical downside correlations ρ_{θ}^{-} and then obtain the BV implied downside beta using Equations 4.

3 | DATA DESCRIPTION AND SAMPLE STATISTICS

3.1 | Data

We employ daily options on the S&P 500 index and its constituents from OptionMetrics for the period from January 1996 to April 2016, a total of 5,116 trading days. We extract the security ID, expiration date, call or put identifier, strike price, best bid, best offer, and implied volatility from the option price file. The sample includes both European and American options. For European options, implied volatilities are calculated using mid-quotes and the Black–Scholes formula. For American options, a binomial tree approach that takes into account the early exercise premium is used. The S&P 500 index serves as a proxy for the U.S. market. The constituents of the S&P 500 index and financial statement data are obtained from COMPUSTAT. The sample period is from January 1995 to April 2016.[4] Daily stock return data are obtained from the center for research in security prices (CRSP). There are a total of 1,052 firms with both option and stock data over the sample period due to the inclusion and deletion of stocks in the S&P 500. The treasury bill yield, which is used as a proxy for the risk-free interest rate, is obtained from the CRSP treasury database.

As in Bakshi, Cao, and Chen (1997), Bakshi, Kapadia, and Madan (2003), Jiang and Tian (2005), and Chang et al. (2011), we use the average of the bid and ask quotes for each option contract. We filter out average quotes

[3]It is not possible to use the CCJV approach to estimate downside beta. The CCJV beta is constructed from option-implied volatility and skewness (see Equation 3. Modeling option-implied volatility or skewness does not require the use of historical stock returns, which are needed to model downside betas or correlations.

[4]The sample periods for the stock and financial statement data are longer than that for the option data, because we use the data in 1995 to form betas and sort stocks in the first month of 1996.

<\$3/8. We also filter out quotes that do not satisfy standard no-arbitrage conditions. For calls, we require the bid price to be less than the spot price and the offer price to be at least as large as the spot price minus the strike price. For puts, we require the bid price to be less than the strike price and the offer price to be at least as large as the strike price minus the spot price. We eliminate in-the-money options because they are less liquid than out-of-the-money (OTM) and at-the-money options. We mitigate the effect of an early exercise premium on our estimations by eliminating put options with $K/S \geq 1.03$ and call options with $K/S \leq 0.97$, where K is the strike price and S is the stock price.

3.2 | Option-implied moments and betas

Option-implied moments are extracted from option data with the model-free approach. We follow the formula of Bakshi et al. (2003), which is described in the Appendix. Moments are computed by integrating over moneyness. To calculate the integrals in the formulas precisely, we require a continuum of option prices. In practice, we do not have these, and we therefore have to approximate them from available option data. As in Carr and Wu (2008) and Jiang and Tian (2005), for each maturity, we interpolate implied volatilities using a cubic interpolation across moneyness levels (K/S) to obtain a continuum of implied volatilities. The cubic interpolation is only effective for interpolating between the maximum and minimum available strike price. For moneyness levels below or above the available moneyness level in the market, we simply extrapolate the implied volatility of the lowest or highest available strike price. We extract a fine grid of 1,000 implied volatilities for moneyness levels between 1/3 and 3. We then convert these implied volatilities into call and put option prices based on the following rule: Moneyness levels (K/S) < 100% are used to generate put prices and moneyness levels ($K/S \geq 1$) greater than 100% are used to generate call prices. This fine grid of option prices is then used to compute the option-implied moments by approximating the Quad and Cubic contracts shown in Appendix using trapezoidal numerical integration. It is important to note that this procedure does not assume that the Black–Scholes model correctly prices options. It merely provides a translation between option prices and implied volatilities.

For each day, we calculate risk-neutral moments using options with different maturities. We require that a minimum of two OTM calls and two OTM puts have valid prices. We use linear interpolation to obtain the 180-day option-implied variance and skewness, using both contracts with maturities of more than 180 days and those with maturities of <180 days.

Panel A of Table 1 presents summary statistics for option-implied volatility and skewness for the sample period from January 1996 to April 2016. The average S&P 500 index volatility is 0.23 and the average stock volatility is 0.37. The average S&P 500 index skewness is −1.66, which is more negative than the average stock skewness of −0.50. This shows that the distribution of both index and stock returns is negatively skewed. Overall, the risk-neutral distribution of index returns is more skewed than the risk-neutral distribution of individual stock returns.

Panel B of Table 1 presents summary statistics for the four measures of standard beta. We find that, for all four measures, the mean value of beta is around unity. The mean weighted market beta theoretically must be equal to 1. The mean historical beta is 1.02, the mean FGK beta is 0.84, the average CCJV beta is 0.88, while the mean BV beta is 1.07. Thus, the historical and BV beta measures are almost unbiased, but the FGK and CCJV betas appear to be biased downwards. The median historical, FGK, and CCJV betas is <1, while the median BV beta is around 1. Panel C reports summary statistics for the three measures of downside beta. The mean historical downside beta is 1.02, the mean BV downside beta is 1.09, while the mean FGK downside beta is 0.65. We expect that the estimation errors of different beta measures will lead to different beta-return relationship. Panel D of Table 1 reports the correlation coefficients between the different measures of standard and downside beta. Each measure of standard beta is generally highly correlated with the corresponding measure of downside beta. Across the different measures, the FGK has a high correlation with the historical and BV measures. The CCJV measure has a much lower correlation with the other measures.

4 | OPTION-IMPLIED BETAS AND THE CROSS SECTION OF STOCK RETURNS

4.1 | Portfolio analysis

To study the risk-return relationship, we perform the portfolio analysis following the early study of Jensen, Black, and Scholes (1972) and the recent study of Buss and Vilkov (2012). We sort the individual securities in the S&P 500 index into five groups at the end of each month, and separately for each beta method, according to their preranked betas. The

TABLE 1 Descriptive statistics

	Observation	Mean	*SD*	25th	Median	75th
Panel A. Option-implied moments						
S&P 500 volatility	244	0.23	0.07	0.18	0.22	0.27
S&P 500 skewness	244	−1.66	0.53	−2.01	−1.60	−1.25
Stock volatility	98,060	0.37	0.16	0.26	0.33	0.43
Stock skew	98,060	−0.50	0.41	−0.71	−0.48	−0.24
Panel B. General betas						
Historical	125,388	1.02	0.48	0.71	0.96	1.25
FGK	97,456	0.84	0.39	0.59	0.80	1.04
CCJV	98,060	0.88	0.68	0.70	0.93	1.19
BV	95,661	1.07	0.37	0.83	1.02	1.26
Panel C. Downside betas						
Historical	122,125	1.02	0.48	0.71	0.96	1.26
FGK	95,678	0.65	0.35	0.42	0.63	0.85
BV	95,678	1.09	0.35	0.85	1.03	1.27
Panel D. Correlations betas						
	Historical	FGK	CCJV	BV	His D	FGK D
FGK	0.69					
CCJV	0.14	0.32				
BV	0.64	0.65	0.20			
His D	0.92	0.63	0.13	0.60		
FGK D	0.50	0.74	0.24	0.47	0.60	
BV D	0.51	0.53	0.21	0.87	0.55	0.57

Note. The table reports descriptive statistics on monthly risk-neutral volatility and skewness for the S&P 500 index and its constituents, the general beta methods and downside beta methods. The sample period is from January 1996 to April 2016. The table reports the number of observation, mean, standard deviation (*SD*) and the 25th, median and 75th percentiles. Panel A reports summary descriptive on risk-neutral volatility and skewness. Panel B reports summary descriptive on the general beta methods. Panel C presents summary statistics on the downside beta methods. Panel D provides the correlation matrix of different betas.

preranked betas are estimated using previous 180-day (126-trading day) daily returns at the end of month t. These stocks are ranked from low to high on the basis of estimated betas and are assigned to five portfolios with equal numbers of securities in each portfolio; the 20% of the stocks with the smallest betas are assigned to the first portfolio, the 20% of the stocks with the biggest betas are assigned to the fifth portfolio and so on. For each beta methodology, and for each portfolio, we calculate the beta and the value-weighted and equally weighted return in month $t+1$. This procedure is repeated for all months during the sample period from January 1996 to April 2016. Finally, we calculate the average values of beta and returns for each portfolio over all months.

Table 2 provides the results for the beta-sorted quintile portfolios from January 1996 to April 2016. Panel A reports the average portfolio betas and returns sorted on historical beta. For the value-weighted portfolios, the difference between the fifth and first portfolio returns (the high–low return spread) is 0.14% per month, while for the equally weighted portfolios, the return spread is 0.28% per month. This is consistent with Fama and MacBeth (1973) and Jensen et al. (1972), who find evidence in support of a positive risk-return relationship as predicted by the CAPM. However, the positive beta-return relation is not significant at the 5% level for either the value-weighted or equally weighted portfolios. Panel B reports results for the quintile portfolios sorted on the FGK beta. A long-short portfolio buying the stocks in the highest beta quintile and shorting the stocks in the lowest beta quintile produces positive average returns. For the value-weighted portfolios, the average high–low return spread is 0.01% per month, while for the equally weighted portfolios, it is 0.19% per month. Panel C reports results for the quintile portfolios sorted on the CCJV beta. We find that the high–low return spread is negative, but the t statistics for the high–low return spread indicate that the difference is not significant.[5] The results for the portfolios sorted on the basis of the BV beta are shown in Panel D. The

[5]Chang et al. (2011) run a cross-section regression of stock returns on the CCJV betas year by year and find that for some years the slopes of the CCJV beta are negative.

TABLE 2 Portfolio analysis sorted by different beta methods

	Low	2	3	4	High	High–low	MR_p
Panel A. Historical beta							
Beta	0.48	0.75	0.95	1.18	1.72	1.24	–
vw-return	0.67	0.83	0.73	0.91	0.80	0.14	0.49
	(3.04)	(3.44)	(2.41)	(2.69)	(1.57)	(0.31)	–
ew-return	0.82	0.97	1.03	1.15	1.10	0.28	0.21
	(3.60)	(3.46)	(3.11)	(3.02)	(1.95)	(0.59)	–
Panel B. FGK beta							
Beta	0.41	0.64	0.79	0.96	1.32	0.91	–
vw-return	0.68	0.73	0.89	0.81	0.69	0.01	0.29
	(3.00)	(2.77)	(2.96)	(2.17)	(1.24)	(0.02)	–
ew-return	0.79	0.85	1.09	1.20	0.98	0.19	0.52
	(3.27)	(2.94)	(3.13)	(2.90)	(1.69)	(0.41)	–
Panel C. CCJV beta							
Beta	0.00	0.74	0.92	1.10	1.48	1.48	–
vw-return	0.85	0.76	0.81	0.71	0.76	−0.09	0.25
	(3.31)	(2.99)	(2.78)	(1.96)	(1.49)	(−0.25)	–
ew-return	1.23	0.98	1.09	0.85	0.73	−0.51	0.65
	(3.75)	(3.51)	(3.35)	(2.22)	(1.40)	(−1.62)	–
Panel D. BV beta							
Beta	0.66	0.86	1.01	1.19	1.62	0.96	–
vw-return	0.63	0.79	1.09	0.84	1.00	0.37	0.66
	(2.57)	(2.85)	(3.57)	(2.19)	(1.74)	(0.79)	–
ew-return	0.74	0.96	1.11	1.17	1.08	0.35	0.30
	(2.85)	(3.16)	(3.29)	(2.89)	(1.76)	(0.68)	–

Note. The five quintile portfolios are sorted by different betas over the sample period from January 1996 to April 2016. At the end of each month, we sort the stocks into quintiles based on their betas. The first portfolio then contains the stocks with the lowest market beta, while the last portfolio contains the stocks with the highest market beta. We then compute the value-weighted and equally weighted monthly returns over the next month for each quintile portfolio, month and beta methodology. The table reports the time-series average of betas and the value-weighted (vw-return) and equally weighted (ew-return) portfolio returns, as well as the high–low portfolio return spread, separately for each beta methodology. In addition, the table provides Newey-West (1987) t statistics for the high–low spread (shown in parentheses) to test whether the spread is significant or not. It also provides P values, obtained from time-series block bootstrapping, for the Patton and Timmermann (2010) monotonic relation (MR) test. The returns are expressed in percentages.

high–low return spread is 0.37% per month for the value-weighted portfolios and 0.35% per month for the equally weighted portfolios. Neither of these values is statistically significant at conventional levels observed from the Newey and West (1987) t statistics. Overall, the portfolio analysis shows that the BV beta gives the biggest high–low return spread compared with the other beta methods.

We perform a formal monotonic relation (MR) test of the risk-return relation, applying the nonparametric technique of Patton and Timmermann (2010). The results of the MR test, with P values obtained from time-series block bootstrapping, are shown in the last column of Table 2. If the high–low return spread is positive (negative), the null hypothesis of the MR test is that there is no relation or a weakly decreasing (increasing) relation between beta and returns, while the alternative hypothesis is that there is an increasing (decreasing) relation between beta and returns. All MR P values are greater than 10%, suggesting that there is no significant evidence to support the existence of a monotonically increasing relation between beta and returns.

To summarize, Table 2 shows that the relationship between the historical, FGK and BV betas, and stock returns is positive, but not statistically significant. The BV beta gives the biggest value-weighted and equally weighted return spread between the extreme portfolios, which is consistent with the findings of Buss and Vilkov (2012).

Figure 1 shows that all beta methods display a noisy beta-return relation across different quintiles for the value-weighted returns. For the value-weighted portfolios, the return difference between the extreme quintile portfolios is more pronounced for the BV and historical betas than for the FGK and CCJV beta methods. The return spread using BV beta is more pronounced than that using historical beta. The plot of the BV beta and

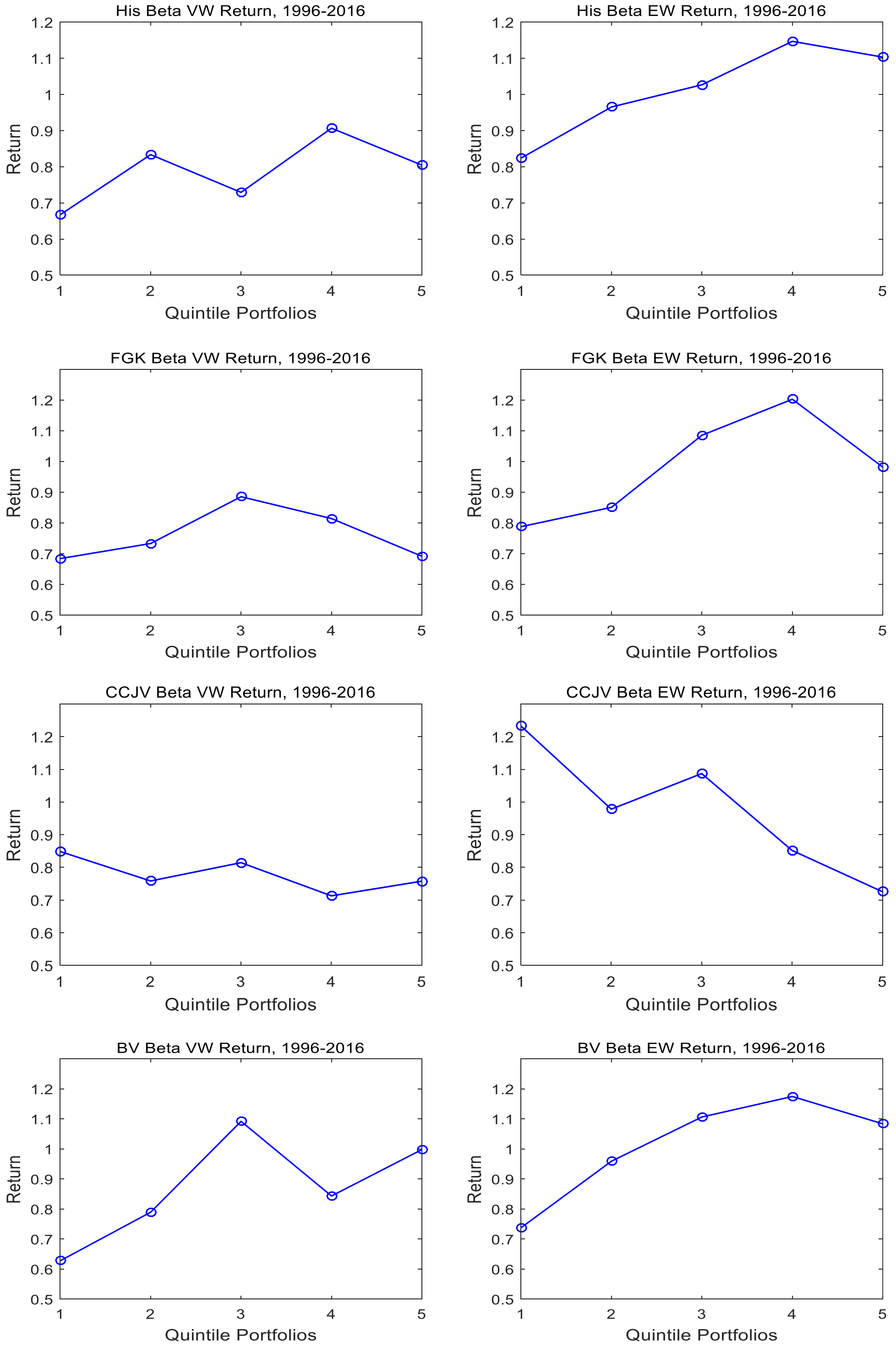

FIGURE 1 Betas and portfolio returns. The figure shows the annualized returns in percentages of the five quintile portfolios sorted by betas over the sample period from January 1996 to April 2016. At the end of each month, we sort the stocks into quintiles based on their betas. The first portfolio then contains the stocks with the lowest beta, while the last portfolio contains the stocks with the highest beta. We then compute the annualized value-weighted and equally weighted returns over the next month for each quintile portfolio, month and beta methodology. Exact numerical values for the betas and returns of each portfolio are shown in Table 2. The four panels present the results for four different beta methods. BV, Buss and Vilkov (2012); CCJV, Chang, Christoffersen, Jacobs, and Vainberg (2011); FGK, French, Groth, and Kolari (1983) [Color figure can be viewed at wileyonlinelibrary.com]

TABLE 3 Fama–MacBeth regressions for general betas

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
Constant	0.77	0.61	1.42	0.37	1.29	1.28	1.47	1.28
	(2.70)	(2.02)	(4.26)	(0.92)	(1.69)	(1.67)	(1.96)	(1.67)
Historical	0.21				−0.05			
	(0.56)				(−0.16)			
FGK		0.62				0.14		
		(1.27)				(0.36)		
CCJV			−0.41				−0.40	
			(−2.28)				(−3.08)	
BV				0.53				0.22
				(1.06)				(0.43)
Size					−0.14	−0.15	−0.11	−0.14
					(−2.28)	(−2.44)	(−1.89)	(−2.32)
BM					0.02	0.02	0.02	0.02
					(0.32)	(0.28)	(0.33)	(0.26)
ivol					2.99	−1.14	6.15	−11.00
					(0.18)	(−0.07)	(0.35)	(−0.60)
VRP					0.06	−0.18	−0.21	−0.19
					(0.06)	(−0.15)	(−0.18)	(−0.13)
Momentum					0.03	0.06	0.19	0.11
					(0.07)	(0.18)	(0.52)	(0.32)
Illiquidity					0.59	0.57	0.35	0.54
					(1.74)	(1.68)	(0.92)	(1.62)
Lag return					−1.54	−1.59	−1.41	−1.62
					(−1.95)	(−2.01)	(−1.64)	(−2.05)
Return_m	−0.35	−0.22	1.51	−0.24				
					(−0.15)	(−0.10)	(0.61)	(−0.10)
Adj R^2 (%)	6.66	6.32	2.01	7.27	16.64	16.67	15.14	16.70

Note. The table shows the results for the Fama and MacBeth (1973) regression of monthly stock returns on betas and firm characteristics. The sample period is from January 1996 to April 2016. We report the average of coefficients and their *t* statistics (shown in parentheses) of the independent variables.

the value-weighted returns shows that the pattern is closest to linear compared with the historical, FGK, and CCJV betas. The equally weighted return for both the historical and BV beta methods displays a monotonically increasing risk-return relation.

4.2 | Fama–MacBeth regressions

We adopt the Fama and MacBeth (1973; FM) regression approach to further examine the risk-return relationship, and in particular, to explore its robustness to a wide range of firm-specific characteristics. The previous literature supports the existence of a firm size effect (Banz, 1981), a book-to-market effect (Basu, 1983), a momentum effect (Jegadeesh & Titman, 1993), an idiosyncratic volatility effect (Ang, Hodrick, Xing, & Zhang, 2006), a reversal effect (Jegadeesh, 1990; Lehmann, 1990), a maximum daily return effect (Bali, Cakici, & Whitelaw, 2011) and an illiquidity effect (Amihud, 2002). The calculation of firm size and book-to-market follows Fama and French (1992). We define the firm size (*size*) as the natural logarithm of the market capitalization from the previous day, where market capitalization is equal to the stock price multiplied by the number of shares outstanding. Book-to-market (*BM*) is the natural logarithm of book value to market value, where book value is the book value of common equity plus balance-sheet deferred taxes. Idiosyncratic volatility (*ivol*) is the standard deviation of the residuals from a regression of the excess stock return on the excess market return and the size (*SMB*) and book-to-market (*HML*) factors of Fama and French (1993), again using daily returns over the previous one year.[6] The momentum measure

[6]The daily market excess return, SMB, HML and the risk-free rate are taken from Kenneth French's website, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/datalibrary.html.

(*Momentum*) is the cumulative daily stock return from month $t-12$ to month $t-1$. The illiquidity measure (*Illiquidity*) is the average of the ratio of the daily absolute return to the (dollar) trading volume over the previous year. Following Jegadeesh (1990) and Lehmann (1990), the reversal measure (*lag return*) is defined as the monthly return over the previous month. The maximum daily return (*return_m*) is defined as the maximum daily return over the previous month. As in Carr and Wu (2008), the variance risk premium (*VRP*) is defined as the difference between realized variance and option-implied variance:

$$VRP(t) = \sigma_P^2(t)\text{-}\sigma_Q^2(t) \tag{11}$$

where $\sigma_P^2(t)$ and $\sigma_Q^2(t)$ denote the realized and implied variances in month t, respectively.

Table 3 presents the results for the Fama and MacBeth (1973) regressions of stock returns on the different measures of beta and firm-specific characteristics. Models 1–4 include only beta in the Fama and MacBeth (1973) regression. When beta is the only independent variable, the coefficients of these betas are not significant except in the case of the CCJV beta, where the coefficient is actually negative. When firm-specific variables are included in the Fama and MacBeth (1973) regressions in Models 5–8, we find that beta still has no significant explanatory power for stock returns except for the CCJV beta, which again has a negative coefficient. Additionally, we find that size and lagged return are significantly and negatively related to stock returns.

TABLE 4 Portfolio analysis on implied downside betas

	Low	2	3	4	High	High–low	MR_p
Panel A. Historical downside beta							
Beta	0.48	0.76	0.96	1.19	1.72	1.24	–
vw-return	0.71	0.80	0.73	0.90	0.86	0.15	0.35
	(3.26)	(3.12)	(2.58)	(2.55)	(1.69)	(0.34)	–
ew-return	0.82	0.97	1.03	1.10	1.16	0.33	0.10
	(3.65)	(3.52)	(3.21)	(2.82)	(2.05)	(0.72)	–
Panel B. FGK downside beta							
Beta	0.24	0.47	0.62	0.77	1.07	0.83	–
vw-return	0.74	0.76	0.71	0.99	0.67	−0.07	0.88
	(3.16)	(2.84)	(2.34)	(2.80)	(1.30)	(−0.18)	–
ew-return	0.83	0.90	1.00	1.21	1.00	0.17	0.57
	(3.42)	(3.01)	(2.87)	(2.96)	(1.82)	(0.41)	–
Panel C. BV downside beta							
Beta	0.70	0.89	1.03	1.21	1.63	0.93	–
vw-return	0.64	0.90	0.86	0.89	1.07	0.43	0.12
	(2.66)	(3.33)	(2.71)	(2.27)	(1.69)	(0.78)	–
ew-return	0.79	0.92	1.09	1.06	1.19	0.39	0.27
	(3.25)	(3.07)	(3.28)	(2.54)	(1.92)	(0.76)	–

Note. The five quintile portfolios are sorted by downside betas over the sample period from January 1996 to April 2016. At the end of each month, we sort the stocks into quintiles based on their downside betas. The first portfolio then contains the stocks with the lowest market downside beta, while the last portfolio contains the stocks with the highest market downside beta. We then compute the value-weighted and equally weighted monthly returns over the next month. The table reports the time-series averages of the value-weighted (vw-return) and equally weighted (ew-return) portfolio returns, as well as the high–low portfolio return spread, separately for each downside beta methodology. In addition, the table provides Newey and West (1987) t statistics for the high–low spread (shown in parentheses) to test whether the spread is significant or not. It also provides P values, obtained from time-series block bootstrapping, for the Patton and Timmermann (2010) MR test. The returns are expressed in percentages.

5 | OPTION-IMPLIED DOWNSIDE BETAS AND THE CROSS SECTION OF STOCK RETURNS

5.1 | Portfolio analysis

We sort the individual securities in the S&P 500 index into five groups at the end of each month by each of the four measures of downside beta. Portfolio 1 includes firms with the lowest downside betas and portfolio 5 contains firms

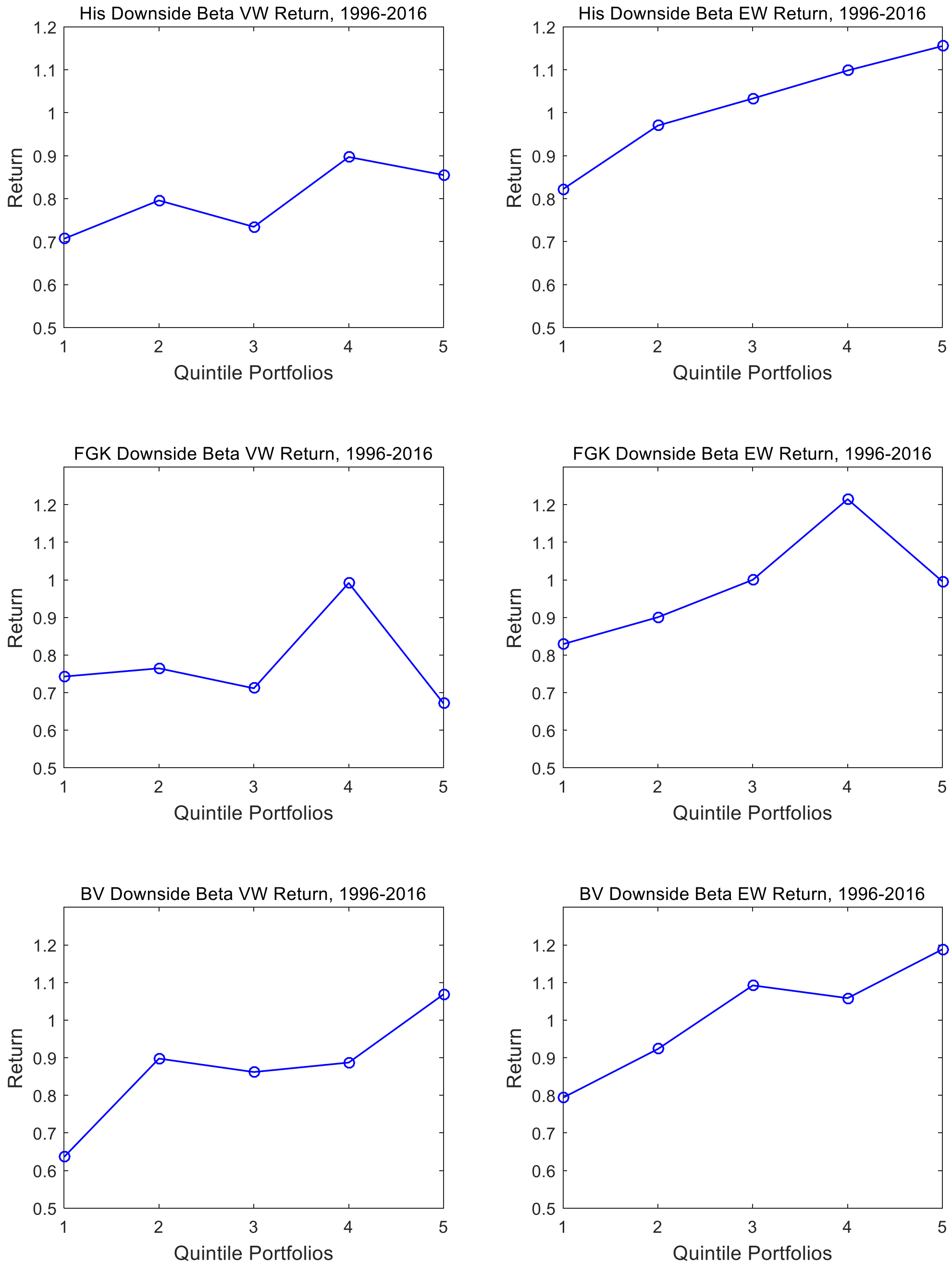

FIGURE 2 Downside betas and portfolio returns. The figure shows the annualized returns in percentages of the five quintile portfolios sorted by downside betas over the sample period from January 1996 to April 2016. At the end of each month, we sort the stocks into quintiles based on their downside betas. The first portfolio contains the stocks with the lowest downside market beta, while the last portfolio contains the stocks with the highest downside market beta. We then compute the annualized value-weighted and equally weighted returns over the next month for each quintile portfolio, month and downside beta methodology. Exact numerical values for the returns and betas of each portfolio are shown in Table 4. The three panels present the results for different downside beta methods. BV, Buss and Vilkov (2012); CCJV, Chang, Christoffersen, Jacobs, and Vainberg (2011); FGK, French, Groth, and Kolari (1983) [Color figure can be viewed at wileyonlinelibrary.com]

with the highest downside betas. We then calculate the annualized value-weighted and equally weighted return for each beta method, for each portfolio in the next month. The procedure is repeated for all months. Table 4 provides a summary of the results. The table shows that the high–low return spread (the difference between the fifth and first portfolio returns) is positive for the historical, FGK, and BV downside beta methods in most cases. Taking the value-weighted returns as an example, the high–low return spread is 0.15% per month for the historical downside beta in Panel A, −0.07% per month for the FGK downside beta in Panel B and 0.43% per month for the BV downside beta in Panel C. The portfolio sorting method thus suggests that there is a positive relationship between the historical and BV downside betas, and stock returns, although in neither case is the difference statistically significant at conventional levels. As in the case of standard beta, the BV implied downside beta gives the biggest value-weighted and equally weighted high–low return spread between the extreme portfolios. From the MR test in the last column of Table 4, we find that all MR *P* values are greater than 10% for the value-weighted returns, suggesting that there is no monotonically increasing relation between downside beta and either value-weighted or equal-weighted returns.

Comparing the results in Table 4 with those in Table 2, we see that the BV implied downside beta performs better than the BV standard beta in terms of the positive and linear beta-return relation. More specifically, the BV downside beta gives an average high–low return spread of 0.43% per month for value-weighted returns, compared with a difference of 0.37% per month for the BV standard beta. The result that the BV downside beta outperforms the BV standard beta is consistent with published research (e.g., Ang et al., 2006; Post & Van Vliet, 2004). For instance, Post and Van Vliet (2004) find that the mean semivariance CAPM strongly outperforms the traditional mean-variance CAPM in terms of its ability to explain the cross section of U.S. stock returns.

TABLE 5 Fama–MacBeth regressions for implied downside beta

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Constant	0.77	0.74	0.48	1.33	1.21	1.23
	(2.74)	(3.04)	(1.13)	(1.76)	(1.62)	(1.60)
His(−)	0.21			−0.11		
	(0.56)			(−0.41)		
FGK(−)		0.52			0.05	
		(1.09)			(0.16)	
BV(−)			0.42			0.13
			(0.80)			(0.25)
Size				−0.13	−0.14	−0.13
				(−2.23)	(−2.28)	(−2.19)
BM				0.03	0.01	0.02
				(0.36)	(0.20)	(0.24)
ivol				3.55	0.45	−8.63
				(0.22)	(0.03)	(−0.45)
VRP				0.18	0.40	0.09
				(0.16)	(0.36)	(0.06)
Momentum				0.02	0.07	0.13
				(0.06)	(0.18)	(0.37)
Illiquidity				0.58	0.48	0.51
				(1.69)	(1.37)	(1.50)
Lag return				−1.61	−1.63	−1.72
				(−2.00)	(−1.97)	(−2.13)
Return max				−0.09	0.50	0.44
				(−0.04)	(0.21)	(0.19)
Adj R^2 (%)	6.44	4.68	7.15	16.46	15.97	16.32

Note. The table shows the results for the Fama and MacBeth (1973) regression of one-month ahead monthly stock returns on downside betas and firm-specific characteristics. The downside betas include the historical beta (His(−)), the FGK downside beta (FGK(−)), and the BV downside beta (BV(−)). The sample period is from January 1996 to April 2016. We report the coefficients and the *t* statistics (shown in parentheses) of the independent variables.
*, **, and *** denote the 10%, 5%, and 1% significance levels, respectively.

Figure 2 shows that the three downside beta methods display a noisy beta-return relation across different quintile portfolios for the value-weighted returns. For the value-weighted portfolios, the return difference between the extreme quintile portfolios is more pronounced for the BV downside beta than for the historical and FGK downside betas. The FGK downside beta gives the flattest beta-return relation, while the BV downside beta displays a relatively increasing risk-return relation. The equally weighted quintile portfolio returns for the historical and BV downside betas display a monotonically increasing risk-return relation.

5.2 | Fama–MacBeth regressions

We run the Fama and MacBeth (1973) monthly regression of stock returns on each of the three measures of downside beta, and include the same set of control variables, namely, firm size, book-to-market ratio, idiosyncratic volatility, the variance risk premium, momentum, lagged return, maximum daily return, and illiquidity. Table 5 presents the results of this analysis. Models 1–3 show the results for the Fama and MacBeth (1973) regression including only downside beta. For all three approaches, the coefficient on downside beta is positive but not significant. When the control variables are added in Models 4–6, the coefficient on downside beta remains insignificant in all three cases. The Fama and MacBeth (1973) regression approach therefore suggests that there is no significant relationship between returns and downside beta.

6 | CONCLUSION

Motivated by the earlier research of Buss and Vilkov (2012), this paper further investigates the relation between option-implied betas and stock returns. Consistent with the findings of Buss and Vilkov (2012), using portfolio analysis, we find that the BV beta outperforms other beta methods, giving the biggest positive high–low return spread. However, the return spread is not statistically significant.

We introduce measures of option-implied downside beta by combining the downside correlation of Ang et al. (2002) and the option-implied moments of Bakshi et al. (2003). When sorting stocks by downside beta, we again find that the BV downside beta yields the biggest high–low return spread. Moreover, a stronger beta-return relation is obtained using the BV downside beta than the BV standard beta.

We further explore the robustness of the beta-return relation using the Fama and MacBeth (1973) regression methodology. We find that using either standard beta or downside beta, the beta-return relationship is not significant nor is it robust to the inclusion of firm characteristics. We, therefore, conclude that there is no robust relation between option-implied beta and stock returns.

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APPENDIX

Estimation of risk-neutral moments

To compute risk-neutral model-free variance and skewness, we follow the formulas in Bakshi and Madan (2000) and Bakshi et al. (2003). Bakshi and Madan (2000) show that the continuum of characteristic functions of risk-neutral return density and the continuum of options are equivalent classes of spanning securities. Any payoff function with bounded expectation can be spanned by out-of-the-money (OTM) European calls and puts. Based on this insight, Bakshi et al. (2003) formalize a mechanism to extract the variance and skewness of the risk-neutral return density from a contemporaneous collection of OTM calls and puts. Their method relies on a continuum of strikes and does not incorporate specific assumptions about an underlying model. The two moments can be expressed as functions of payoffs on a quadratic and a cubic contract.

The prices of the quadratic and cubic contracts are given by

$$\text{Quad} = \int_S^\infty \frac{2\left(1-\ln\left(\frac{K}{S}\right)\right)}{K^2} C(\tau,K)\, dK + \int_0^S \frac{2\left(1+\ln\left(\frac{S}{K}\right)\right)}{K^2} P(\tau,K) dK, \tag{A.1}$$

$$\text{Cubic} = \int_S^\infty \frac{6\ln\left(\frac{K}{S}\right) - 3\left(\ln\left(\frac{K}{S}\right)\right)^2}{K^2} C(\tau,K)\, dK - \int_0^S \frac{6\ln\left(\frac{K}{S}\right)+3\left(\ln\left(\frac{K}{S}\right)\right)^2}{K^2} P(\tau,K) dK, \tag{A.2}$$

where S and K are the underlying stock price and strike price, respectively, and C and P are the call and put prices, respectively.

Using the prices of these contracts, the risk-neutral moments can be calculated as:

$$\text{VAR} = e^{r\tau}\, \text{Quad} - \mu^2, \tag{A.3}$$

$$\text{SKEW} = \frac{e^{r\tau}\text{Cubic} - 3\mu\, e^{r\tau}\text{Quad} + 2\mu^3}{\text{VAR}^{3/2}}, \tag{A.4}$$

where $\mu = e^{r\tau} - 1 - \frac{e^{r\tau}\text{Quad}}{2} - \frac{e^{r\tau}\text{Cubic}}{6}$.