Variance Risk Premia, Asset Predictability Puzzles, and Macroeconomic Uncertainty*

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Abstract

This paper presents predictability evidence of the implied-expected variance difference, or variance risk premium, for financial market risk premia: (1) the variance difference measure predicts a *positive* risk premium across equity, bond, currency, and credit markets; (2) such a *short-run* forecastability peaks at one month horizon and dies out as horizon rises; (3) the short-run predictability is *complementary* to that of the standard predictor variables—P/E ratio, forward rate, interest differential, and short rate. These findings are justifiable by a general equilibrium model that incorporates stochastic economic uncertainty and recursive utility function. Within such a framework, the *negative* volatility risk premium implied from option prices is internally consistent with the *positive* variance risk premium embedded in underlying assets.

JEL classification: G12, G13, G14.
Keywords: Macroeconomic uncertainty, asset return predictability, variance risk premia, recursive utility function.

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Abstract

This paper presents predictability evidence of the implied-expected variance difference, or variance risk premium, for financial market risk premia: (1) the variance difference measure predicts a *positive* risk premium across equity, bond, currency, and credit markets; (2) such a *short-run* forecastability peaks at one month horizon and dies out as horizon rises; (3) the short-run predictability is *complementary* to that of the standard predictor variables—P/E ratio, forward rate, interest differential, and short rate. These findings are justifiable by a general equilibrium model that incorporates stochastic economic uncertainty and recursive utility function. Within such a framework, the *negative* volatility risk premium implied from option prices is internally consistent with the *positive* variance risk premium embedded in underlying assets.

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1 Introduction

Option implied volatility, such as the Chicago Board Option Exchange's VIX index, is widely viewed by investors as the *market gauge of fear* (Whaley, 2000).[1] In related academic research, the difference between the implied and expected volatilities has been interpreted as an indicator of the representative agent's risk aversion (Rosenberg and Engle, 2002; Bakshi and Madan, 2006; Bollerslev, Gibson, and Zhou, 2008). An alternative interpretation is that the implied-expected variance difference, as a proxy for variance risk premium, is due to the macroeconomic uncertainty risk (Bollerslev, Tauchen, and Zhou, 2009; Drechsler and Yaron, 2008).

Such an approach relies on the non-standard recursive utility framework of Epstein and Zin (1991) and Weil (1989), such that the consumption uncertainty risk commands a time-varying risk premium. This method follows the spirit of the long-run risks (LRR) models as pioneered by Bansal and Yaron (2004), but focuses on the stochastic consumption volatility as a primary source of financial market risk premia. As demonstrated by Beeler and Campbell (2009), the more recently calibration setting of the long-run risk model in Bansal, Kiku, and Yaron (2007) emphasizes more on the persistent volatility channel as opposed to the persistent long-run growth channel. However, the approach taken here completely shuts down the long-run risk channel.

This paper demonstrates that the difference between implied and expected variances, as a measure for variance risk premium, provides a significant predictability for short-run equity returns, bond returns, forward premiums, and credit spreads. The documented return predictability peaks around one month horizon across these markets, and then dies out as the forecasting horizon increases. More importantly, such a short-term forecastability of financial market risk premia is complementary to the usual established predictor—P/E ratio, forward rate, interest rate differential, and short rate level; in that when combined together, the statistical significance of the variance risk premium proxy is rather increased, instead

[1] In the final quarter of 2008, the VIX index has closed above 50 percent for almost twelve weeks and peaked around 90 percent. As reported by the Wall Street Journal on November 12, 2008, if market volatility continues to remain above 50 percent for just over five weeks, it would have surpassed the **Great Depression**; and such a high volatility signifies all those unknowns that are a greater cloud of what we call **Uncertainty**.

of being crowded out by the standard prediction variable. This constitutes an important evidence that risk premia across major financial markets comove in short-term, and such a common component seems to be predictable by the variance risk premium, as measured by the implied-expected variance difference.

This type of common *short-run* risk factor may be a proxy for stochastic economic uncertainty or consumption volatility risk that varies independently with the consumption growth rate—the latter being the main focus of the *long-run* risk models (Bansal and Yaron, 2004). These empirical results may be consistent with a self-contained general equilibrium model incorporating the effects of such a time-varying economic uncertainty, where the uncertainty risk is priced only under the recursive utility function. Calibration evidence shows that the short-run predictability of equity and bond markets can be qualitatively replicated as in Bollerslev, Tauchen, and Zhou (2009), although the magnitude of predictability from the calibration exercise falls short of what one can achieve with the empirical data. Extensions to currency and credit markets are also plausible for explaining the short-run predictability in these asset prices.

There is a fundamental link between the notion of volatility risk premium implied from option prices and the notion of variance risk premium embedded in equity, bond, currency, and credit markets. Under the arbitrage pricing framework, stochastic volatility of equity market can only be priced, if its innovation is correlated with the market return innovation (Heston, 1993; Bates, 1996; Bakshi and Kapadia, 2003). There is a great deal of evidence that market volatility is negatively correlated with the market returns, such that it provides a hedging service for investors; therefore the *negative* risk premium for stochastic volatility is not only a robust finding of the vast empirical option pricing literature, but also is consistent with the intertemporal hedging argument (Merton, 1973).

However, in order to justify the existence of a variance risk premium embedded in equity, bond, currency, and credit markets in a consumption based asset pricing model, without assuming any statistical correlation between consumption volatility and consumption growth innovations, one need to endow the economic agents with a preference for an earlier resolution of economic uncertainty under the Epstein-Zin recursive utility function (Tauchen, 2005).

Under such a framework, the variance risk premium embedded in financial markets must be positive, as high risk should be compensated by high return. Also under such a framework, the difference between implied and realized variances has the same sign as the variance risk premium embedded in the underlying assets; and both load on the same risk factor—stochastic consumption volatility. In other words, the variance risk premium under the model of stochastic macroeconomic uncertainty must be *positive* by construction, and this is completely consistent with the *negative* volatility risk premium implied from the option prices.

Economic uncertainty and its impact on asset pricing can be examined with other techniques under the recursive preference structure. Bansal and Shaliastovich (2008a) introduce learning and information uncertainty into the long-run risk model, such that endogenously asset returns requires a compensation for jump risks. Chen and Pakos (2008) model the endowment growth rate as a Markov switching process with a constant volatility, where learning brings about an *endogenous* uncertainty premium. On the other hand, Drechsler (2008) applies the Knightian uncertainty about model misspecification with realistic asset dynamics to explain the observed option pricing puzzles. Lettau, Ludvigson, and Wachter (2008) use a Markov switching learning model to describe the long swing changes in constant consumption volatility—Great Moderation—and to draw implications for the declining equity risk premiums.[2]

In contrast, the approach taken here shuts down the the long-run component in consumption growth, and attributes the higher order time-variation in financial market risk premia to the stochastic economic uncertainty as a priced fundamental risk factor. This paper attempts to model the economic uncertainty as a rich consumption volatility dynamics and relies on the insight that derivative prices can be used to pin down the underlying economic uncertainty.

The rest of the paper will be organized as the following, the next section defines the variance risk premium and describe its empirical measurement; Section 3 presents the main

[2]Pástor and Stambaugh (2009) use the uncertainties of expected future and current returns to argue that long-run stock returns are indeed more volatile.

empirical evidence of the short-run predictability on implied-expected variance difference for risk premia on equity, bond, currency, and credit markets; the following section discusses the general equilibrium model of stochastic economic uncertainty and provides calibration implications for the short-run asset predictability puzzles; and Section 5 concludes.

2 Variance Risk Premium and Empirical Measurement

The central empirical finding of this paper is that market risk premia have a common short-run component—variance risk premium—that is not directly observable. However, an empirical proxy can be constructed from the difference between model-free option-implied variance and the conditional expectation of model-free realized variance.

2.1 Variance Risk Premium: Definition and Measurement

To formally define the procedure in quantifying the model-free implied variance, let $C_t(T, K)$ denote the price of a European call option maturing at time T with strike price K, and $B(t, T)$ denote the price of a time t zero-coupon bond maturing at time T. As shown by Carr and Madan (1998), Demeterfi, Derman, Kamal, and Zou (1999) and Britten-Jones and Neuberger (2000), the market's risk-neutral expectation of the return variance between time t and $t+1$ conditional on time t information, or the implied variance $IV_{t,t+1}$, may then be expressed in a "model-free" fashion as the following portfolio of European calls,

$$IV_{t,t+1} \equiv \mathrm{E}_t^Q\left(\mathrm{Var}_{t,t+1}\right) = 2\int_0^{\infty} \frac{C_t\left(t+1, \frac{K}{B(t,t+1)}\right) - C_t\left(t, K\right)}{K^2}\, dK, \tag{1}$$

which relies on an ever increasing number of calls with strikes spanning zero to infinity.[3]

This equation follows directly from the classical result in Breeden and Litzenberger (1978), that the second derivative of the option call price with respect to strike equals the risk-neutral density, such that all risk neutral moments payoff can be replicated by the basic option prices (Bakshi and Madan, 2000). In practice, of course, $IV_{t,t+1}$ must be constructed on the basis of a finite number of strikes. Fortunately, even with relatively few

[3]Such a characterization abstracts from the realistic economic environment that allows for (1) lumpy dividend payment, (2) stochastic interest rate, (3) underlying asset jumps, and (4) limited number and range of option strikes—discretization and truncation errors. See Jiang and Tian (2005) for detailed discussions.

different option strikes this tend to provide a fairly accurate approximation to the true (unobserved) risk-neutral expectation of the future market variance, and, in particular, a much better approximation than the one based on inversion of the standard Black-Scholes formula with close to at-the-money option(s) (Jiang and Tian, 2005; Carr and Wu, 2008; Bollerslev, Gibson, and Zhou, 2008).

In order to define the measure in quantifying the actual return variation, let p_t denote the logarithmic price of the asset. The realized variance over the discrete t to $t+1$ time interval may then be measured in a "model-free" fashion by

$$RV_{t,t+1} \equiv \sum_{j=1}^{n} \left[p_{t+\frac{j}{n}} - p_{t+\frac{j-1}{n}} \right]^2 \longrightarrow \mathrm{Var}_{t,t+1}, \tag{2}$$

where the convergence relies on $n \to \infty$; i.e., an increasing number of within period price observations. As demonstrated in the literature (see, e.g., Andersen, Bollerslev, Diebold, and Ebens, 2001; Barndorff-Nielsen and Shephard, 2002; Meddahi, 2002), this "model-free" realized variance measure based on high-frequency intraday data affords much more accurate ex-post observations of the true (unobserved) return variation than do the more traditional sample variances based on daily or coarser frequency returns. In practice, various market microstructure frictions invariably limit the highest sampling frequency that may be used in reliably estimating $RV_{t,t+1}$.

The variance risk premium is defined as the difference between the ex-ante risk neutral expectation of the future return variance and the objective or statistical expectation of the return variance over the $[t, t+1]$ time interval,

$$VRP_t \;\equiv\; \mathrm{E}_t^Q \left(\mathrm{Var}_{t,t+1} \right) \;-\; \mathrm{E}_t^P \left(\mathrm{Var}_{t,t+1} \right), \tag{3}$$

which is not directly observable in empirical exercises. To construct an empirical proxy for such a variance risk premium concept (3), one need to estimate various reduced-form counterparts of the risk neutral and physical expectations,

$$\widehat{VRP}_t \;\equiv\; \widehat{\mathrm{E}}_t^Q \left(\mathrm{Var}_{t,t+1} \right) \;-\; \widehat{\mathrm{E}}_t^P \left(\mathrm{Var}_{t,t+1} \right). \tag{4}$$

In practice, the risk-neutral expectation $\widehat{\mathrm{E}}_t^Q \left(\mathrm{Var}_{t,t+1} \right)$ is commonly replaced by the CBOE implied variance or VIX^2 and the true variance $\mathrm{Var}_{t,t+1}$ is replaced by its discretized realization $RV_{t,t+1}$. Such an approach is advocated in Bollerslev and Zhou (2006), but the method for constructing the physical expectation $\widehat{\mathrm{E}}_t^P(\cdot)$ is not unique in the literature.

One approach is to estimate a reduced-form multi-frequency auto-regression with potentially multiple lags for $\widehat{\mathrm{E}}_t^P(RV_{t,t+1})$ (Bollerslev, Tauchen, and Zhou, 2009). Of course, for estimating more specific structural jump-diffusion processes, one could use the model-implied objective expectation (Todorov, 2009). To test some specific hypotheses that the variance risk premium should explain risk premia variations across financial markets, one could substitute the conditional expectation $\widehat{\mathrm{E}}_t^P(RV_{t,t+1})$ with various forecasted value or even just the ex post realization (Drechsler and Yaron, 2008). For forecasting purposes only, an alternative approach is to use time-t realized variance $RV_{t-1,t}$ (Bollerslev, Tauchen, and Zhou, 2009), which ensures that the variance risk premium proxy for predicting various risk premia is in the time t information set and would be a correct choice if the realized variance process were unit-root. However, such an empirical *model-free* proxy has a disadvantage of not being guaranteed to be positive. Finally, one could just use a moving average estimate of $\widehat{\mathrm{E}}_t^P(RV_{t,t+1})$, say with a twelve lag, such that no parameters need to be estimated and that the predictor variable is within the current information set. When presenting the empirical findings, I will focus on the method that uses the twelve lag auto-regressive estimate of $\widehat{\mathrm{E}}_t^P(RV_{t,t+1})$, similar to the one used in Bollerslev, Tauchen, and Zhou (2009), while the results based on other methods are available upon request.

2.2 Data Description and Summary Statistics

For the option-implied variance or risk-neutral expectation of return variance, I use the monthly data for the Chicago Board of Options Exchange (CBOE) volatility index VIX[2]. The VIX index is based on the highly liquid S&P500 index options along with the "model-free" approach discussed above explicitly tailored to replicate the risk-neutral variance of a fixed 30-day maturity.[4] The VIX index is invariably subject to some approximation error (see, e.g., the discussion in Jiang and Tian, 2007), but the CBOE procedure for calculating

[4]The CBOE replaced the "old" VIX index, based on S&P100 options and Black-Scholes implied volatilities, with the "new" VIX index, based on S&P500 options and "model-free" implied volatilities, in September 2003.

the VIX has arguably emerged as the industry standard. Thus, in order to facilitate replication and comparison with other studies, we purposely rely on the readily available squared VIX index as our measure for the risk-neutral expected variance.

The intraday data for the S&P500 composite index used in the construction of our "model-free" RV_t measure is provided by the Institute of Financial Markets. Note that a host of practical market microstructure features, including price discreteness, bid-ask spreads, and non-synchronous trading effects, imply that the underlying semi-martingale assumption for the returns is violated at the very highest sampling frequencies. Following the literature, we base our monthly realized variance measure for the S&P500 on the summation of the 78 within day five-minute squared returns covering the normal trading hours from 9:30am to 4:00pm plus the close-to-open overnight return.[5] For a typical month with 22 trading days, this leaves us with a total of $n = 22 \times 78 = 1,716$ "five-minute" returns.

In addition to the variance risk premium, we consider a set of four market risk premium measures with some traditional predictor variables. Specifically, we obtain monthly P/E ratios and index returns for the S&P500 directly from Standard & Poor's, bond returns and forward rates from the monthly CRSP Fama-Bliss data set of 1 to 6 month maturities, forward implied interest rate differentials and spot exchange rates of major currencies (Euro/USD, JPY/USD, GBP/USD) from Bloomberg, and Moody's AAA and BAA corporate bond spreads with Fama-Bliss risk-free interest rate (CRSP). The empirical analysis here is based on the sample period from January 1990 through December 2008, when the new VIX index based on S&P500 index becomes available, except for the exchange rates where the starting date is January 1999 when the Euro came into existence.

To give a visual illustration, Figure 1 plots the monthly time series of variance risk premium, implied variance, and realized variance.[6] The variance risk premium proxy is moderately high during 1990 and 2001 recessions, and much higher around 1997-1998 Asia-Russia-LTCM crisis and 2002-2003 corporate accounting scandals. The huge spike of the

[5]A number of studies, using the volatility signature plot first proposed by Andersen, Bollerslev, Diebold, and Labys (2000), suggest that for highly liquid assets, such as the S&P500 index analyzed here, a five-minute sampling frequency provides a reasonable choice (see, e.g., the discussion in Hansen and Lunde, 2006).

[6]The data series of implied variance (end-of-the-month observation) and realized variance (summation over the month) can be downloaded from `http://sites.google.com/site/haozhouspersonalhomepage/`.

variance risk premium during October 2008 actually leads to an equity market bottom around March 2009. As shown by the empirical results in following section, such a turmoil period in effect strengthens the (positive) forecastability of variance risk premium for various market risk premia. Both implied and realized variance series show similar patterns, except that the shot up of volatilities during October 2008 already surpasses the initial shock of October 1929 during the Great Depression.

Table 1 Panel A compares the summary statistics of different variance risk premium proxies based on alternative ways to estimate the conditional expectation of realized variances, which are similar but more comprehensive than those reported by Bollerslev and Zhou (2006) and Drechsler and Yaron (2008). The mean level of variance risk premium is around 17 to 22 (in percentage-squared, not annualized) across five different estimates, with a standard deviation around 22 to 28. Not surprisingly, the variance risk premiums based on current and lagged realized variance have the highest kurtosis of 44 to 46, while the full sample AR (12) estimate has the lowest of 17. Also noteworthy is that the variance risk premium estimates based on raw current and lag realized variance has a skewness of -3, while others are all positive skewed. The negative skewness is entirely driven by the one observation of negative spike in October 2008 (Figure 2 upper two panels) and by not using the *forecasted* realized variance in constructing the variance risk premium (Figure 2 lower two panels). Finally the auto-regressive coefficient of order one is generally low between 0.26 and 0.76, with the full sample AR (12) achieves the lowest value. Figure 2 shows the variance risk premia based on other four estimates of the expected realized variance, where the recursive AR(12) and MA(12) approaches both suggest that the variance risk premium had achieved the unprecedented historical height during November 2008.

Basic summary statistics for the monthly returns and predictor variables are given in Table 1 Panels B to E. The mean excess return on the S&P500 over the sample period equals 4.65 percent annually, reflecting the significantly lowered market returns during the 2007-2008 financial crisis and economic downturn. The one month holding period returns for 3-month and 6-month t-bills are 4.26 and 4.66 percent annualized, while the monthly exchange returns are essentially zero for Euro, Pound, and Yen. Finally, the credit spread for Moody's AAA rating is 1.25 percent and BAA 2.14 percent. The sample means for the implied-realized variances is about 18.30 (in percentages squared). The numbers for the traditional long-run forecasting variables—P/E ratio, forward rate, interest differential, and short rate—are all directly in line with those reported in previous studies. In particular, all of these variables are highly persistent with first order autocorrelations ranging from 0.97 to 1.01. In contrast, the serial correlation in the variance risk premium proxy only equals 0.26. As such, this alleviates one of the common concerns related to the use of highly persistent predictor variables and the possibility of spurious or unbalanced regressions.[7]

3 Short-Run Predictability Puzzles of Financial Assets

The difference between implied and expected variances, or variance risk premium, provides a significant predictability for equity returns, bond returns, forward premiums, and credit spreads. The documented return predictability peaks around one month, and then dies out as the forecasting horizon increases. More importantly, such a short-term forecastability is complementary to the standard predictor variables—like P/E ratio, forward rate, interest rate differential, and short rate level; in that when combined together, the economic and statistical significance of the variance risk premium proxy are preserved at least and most often increased.

The forecasts are based on linear regressions of asset returns or spreads on a set of lagged predictor variables, including the proxy for variance risk premium. Data are monthly observations with horizons up to one or two years. All of the reported t-statistics are based on heteroskedasticity and serial correlation consistent standard errors (Newey and West, 1987). The discussion focuses on the estimated slope coefficients and their statistical significance as determined by the robust t-statistics. The forecasts accuracy of the regressions are also measured by the corresponding adjusted R^2's. However, as previously noted, for the highly

[7]Inference issues related to the use of highly persistent predictor variables have been studied extensively in the literature, see, e.g., Stambaugh (1999), Ferson, Sarkissian, and Simin (2003), Lewellen (2004), and Campbell and Yogo (2006) and the references therein. Some studies, e.g., Boudoukh, Richardson, and Whitelaw (2008) and Goyal and Welch (2003, 2008) have argued that the apparent predictability may be attributed to using of strongly serially correlated predictor variables with or without overlapping data.

persistent predictor variables—not necessarily the case of variance risk premium—the R^2's for the overlapping multi-period return regressions need to be interpreted with great caution (Boudoukh, Richardson, and Whitelaw, 2008).

3.1 Equity

Building on the the results reported in Bollerslev and Zhou (2007) and Bollerslev, Tauchen, and Zhou (2009), here I focus on the regression of S&P500 returns on a long-run predictor—P/E ratio and a short-run predictor—variance risk premium,

$$xr_{t+h} \;=\; b_0(h) \;+\; b_1(h)\, VRP_t \;+\; b_2(h)\; \log\left(P_t/E_t\right) \;+\; u_{t+h,t}, \tag{5}$$

where xr_{t+h} is the horizon-scaled market excess return and the horizon h goes out to 24 months. Both univariate and bivariate regressions results are reported.

Table 2 top row shows that the degree of predictability, offered by the variance risk premium VRP_t , starts out fairly high at the monthly horizon with an R^2 of 2.55 percent. While the robust t-statistic for testing the estimated slope coefficient associated with the variance difference VRP_t greater than zero is the highest among all horizons at 3.28. The quarterly return regression results in a impressive t-statistic of 2.98 and achieves the highest corresponding R^2 of 5.86 percent. The t-statistic remains marginally significant at the 18-month horizon, and then gradually taper off for longer return horizons.

On the other hand, as shown in the middle row in Table 2, the usual long-run predictor $\log P_t/E_t$ ratio starts out barely significant at 10 percent level from one to nine months, and then from one year to two year it becomes marginally significant at 5 percent level, with t-statistic -2.04 and R^2 20.22 percent towards the end. Many of the empirical studies (see, e.g., Campbell and Shiller, 1988; Lettau and Ludvigson, 2001; Ang and Bekaert, 2007, among others) have argued that the degree of predictability afforded by the different valuation ratios tend to be the strongest over longer multi-year horizons. However, the conventional t-statistic and/or the R^2's with highly persistent predictor variables and overlapping returns may by construction increase proportionally with the return horizon and the length of the overlap.

Turning to the joint regressions reported in the bottom row of the Table 2, it is clear that combining the variance premium with the P/E ratio results in an even higher R^2 of 4.39 percent at monthly horizon, which is higher than the sum of two R^2's in the respective univariate regressions. The t-statistics for VRP_t and $\log P_t/E_t$ are also impressive, 2.88 and -1.85, respectively. The predictability of variance premium extends beyond the eighteen month horizon and its significance remains above the 5 percent level. Intuitively, the variance risk premium and the P/E ratio may jointly capture the important short-run and long-run aspects of risk prices embedded in the equity returns. It may be helpful to view these results through an angle of decomposing the long-run and short-run risk prices inside a dynamic asset pricing model.

Taken as a whole, the results in Table 2 reveal a clear pattern in the degree of predictability afforded by the variance risk premium with the largest t-statistic occurring at the monthly horizon. More specifically, the empirical estimates for all of the monthly horizons ranging from one month to two years reported in Figure 3 show that the estimated slope coefficients peak at one month horizon and R^2's peak around three month horizon, and then both decline toward zero. Note that the 95 percent standard error band is bounded bellow from zero up to the horizon of 18 months. In Section 4, I will try to calibrate a consumption-based model with time-varying volatility-of-volatility, to replicate such a striking pattern.

3.2 Bond

The failure of the Expectations Hypothesis (EH) of interest rates can be best characterized as that bond excess return, estimated from forward rates, is largely predictable and time-varying countercyclically (Fama and Bliss, 1987).[8] This is conceptually equivalent to regressing bond yield changes on yield spreads and producing a negative slope coefficient instead of unity (Campbell and Shiller, 1991). Here I adopt the conventional forward rate setup but augment

[8]The forward rate regression is recently extended by Cochrane and Piazzesi (2005) to multiple forward rates, by Ludvigson and Ng (2008) to incorporate extracted macroeconomic factors, and by Wright and Zhou (2009) to augment with a realized jump risk measure. However, these studies use 2-5 year zero coupon bonds with a one year holding period, where the variance risk premium variable has a zero forecasting power of the bond risk premia.

it with the variance risk premium variable,

$$xhpr^n_{t+h} \;=\; b^n_0(h) \;+\; b^n_1(h)\; VRP_t \;+\; b^n_2(h)\; \left[f_{t-1}(n-h,h) - y_{t-1}(h)\right] \;+\; u^n_{t+h,t}, \qquad (6)$$

where $xhpr^n_{t+h}$ is the excess holding period return of zero coupon bonds with hold period $h = 1, 2, 3, 4, 5$ month and maturity $n = 2, 3, 4, 5, 6$ month (in excess of the yield on a h-month zero coupon bond); $f_{t-1}(n-h,h)$ is the forward rate for a contract h-month ahead with $n-h$-month length; and $y_{t-1}(h)$ is the h-month zero coupon bond yield.[9]

As shown in Table 3, the variance risk premium can significantly forecast the one month holding period excess returns of the two-six month Fama-Bliss t-bills, with a positive slope coefficient—indicating positive variance risk premium in bond returns—around 0.006 to 0.013. Considering the average level of variance risk premium of 18.30, this magnitude translates to an average bond risk premium induced by variance risk premia around 11 to 24 basis points. More importantly, the Newey-West t-statistics are all well above the 1 percent significance level, with an R^2 around 2.77 to 4.57 percent. Note that the variance risk premium variable has a persistence level of 0.26 in terms of its AR(1) coefficient. Moving to the two month holding period, the t-ratios reduce to a marginal significance of 1.22 to 2.05, and the R^2 decreases to 0.86 to 3.54 percent. The bond return predictability of variance risk premium basically converges to zero as the holding period increase to three-five months.

As Table 4 indicates, the forward rate is indeed a powerful predictor for excess bond returns for two-to-six month bonds with one-to-five month holding periods—t-statistics all above 1 percent level and R^2 between 2.65 and 36.08 percent. Another pattern is that the magnitudes of t-statistics and R^2 are generally higher at the one-month horizon and lower toward the five-month horizon. Overall, the predictability of forward spreads for short-term bills are similar, if not stronger than those reported for long-term bonds (Fama and Bliss, 1987). One should be cautioned the forward rate has a very high degree of persistence level,

[9] Note that in the original Fama and Bliss (1987) regression of 2-5 year zero coupon bonds with 1-4 year holding periods at a monthly sampling frequency, the forward spread $f_t(n-h,h) - y_t(h)$ is at current month t. But they suggested using the lagged forward spread to break the potential first order serial correlation in the market microstructure error, which may artificially inflate the bond return predictability. Such a concern is more relevant to our short-term return predictability regressions. The results based on the current forward spread are similar and available upon request.

with an AR(1) coefficient being 0.97 and 0.99 for the two- and five-month ahead forward rate of one month length (see Table 1 Panel C).

More importantly, when variance risk premium is combined with forward rates, as shown in Table 5, the predictability of the variance risk premium remains intact. This result suggests that the variance difference and forward spread are proxies for different components in bond risk premia—with the forward rate proxing the "long-run" risk factor while the variance difference proxing the "short-run" risk factor.

3.3 Currency

Uncovered interest rate parity (UIP) predicts that the expected *appreciation* of the foreign currency must equal the difference between domestic and foreign interest rates; such that an investor is indifferent between holding a domestic or a foreign bond. However, vast empirical evidence since Fama (1984) have found exact the opposite—an increase in the domestic interest rate corresponds rather a *depreciation* of the foreign currency.[10] The UIP violation is especially challenging at short horizons (Hodrick, 1987), and here I provide evidence that the variance risk premium helps explain the short-run variation in exchange rates.

$$s_{t+h} - s_t \; = \; b_0(h) \; + \; b_1(h) \, VRP_t \; + \; b_2(h) \; [y_t(h) \; - \; y_t^*(h)] \; + \; u_{t+h,t}, \tag{7}$$

where s_t is the log exchange rate—domestic currency over foreign currency, while $y_t(h)$ and $y_t^*(h)$ are h-period zero coupon bond yields home and abroad.

Table 6 reports the result of UIP regressions augmented by the variance risk premium variable for three major currency pairs, Japanese Yen (JPY), British Pound (GBP), and Euro against the US dollar, from 1999 to 2008 when the Euro is in existence. The interest rate differential, middle column of the table, explains at most 1.31 percent of the R^2 of the exchange rate returns at one month horizon, while increases to 25, 5, and 14 percents at twelve month horizon, for Euro, GBP, and JPY respectively. The slope coefficients are on average negative and are mostly indistinguishable from zero. So the rising R^2 with horizons

[10] It should be emphasized that in the long-run the UIP violation is muted (Alexius, 2001), and that consumption-based interpretation tend to explain well the cross section of foreign exchange returns (Lustig and Verdelhan, 2007).

may be driven by the near unit-root property of the interest rate differential, with an AR(1) coefficient ranging 0.98 to 1.01 (Panel D in Table 1); and by using overlapping data in long-horizon regressions.

Turning our attention to the variance risk premium in the left column of Table 6, for both Euro and GBP, the implied-expected variance difference explains about 10 percent of total variation at the monthly horizon, with a similar positive slope coefficient (3.35 and 3.03) and a highly significantly t-statistics (3.73 and 3.14). For JPY, the slope coefficient of variance risk premium is negative but indifferent from zero, with essentially a zero explaining power in term of the R^2.

Interestingly when combined with the interest rate differential, the variance risk premium variable is slightly more significant at one month horizon for Euro with t-statistics being 4.59 and remains the same for GBP with t-statistics being 3.11, while the interest rate differential still has no explaining power at most forecasting horizons. In essence, the strong predictability from the variance risk premium for the Euro and GBP dissipates quickly as the forecasting horizon increases, consistent with the result reported for equity and bond in the earlier subsections.

Such a differential pattern in short-run predictability of exchange rate from the US variance risk premium, for Euro and GBP versus JPY, may be interpreted later in Section 4 as missing an important explanatory variable—the variance risk premium of the foreign markets.

3.4 Credit

The relatively large and time-varying credit spread on corporate bond has long been viewed as an anomaly in the corporate finance literature (Jones, Mason, and Rosenfeld, 1984; Elton, Gruber, Agrawal, and Mann, 2001; Huang and Huang, 2003). Here I provide some new evidence that, in additional to the standard predictor namely the interest rate level (Longstaff and Schwartz, 1995), the variance risk premium proxy also helps to explain the short-run movement in credit spreads.

$$\text{CS}_{t+h} \; = \; b_0(h) \; + \; b_1(h) \, VRP_t \; + \; b_2(h) \, r_{f,t} \; + \; u_{t+h,t}, \tag{8}$$

where the credit spread CS_{t+h} of h month ahead is being forecasted by the short rate $r_{f,t}$ and variance risk premium.

As shown in Table 7, short term interest rate is indeed a predominant predictor of the future credit spread levels, with t-statistics of -3.94 for investment grade (Moody's AAA rating) and -2.94 for speculative grade (Moody's BAA rating). The adjusted R^2 is around 32 percent, and the negative sign of the slope coefficient is consistent with the risk-neutral drift interpretation in Longstaff and Schwartz (1995).[11] Although the significance of the short rate level extends to the six-month horizon, it is a very persistent variable with an AR(1) coefficient of 0.99 (Panel E in Table 1).

Note that if we include the variance risk premium alone in the forecasting regressions, its statistical significance is above 1 percent at one month horizon, with t-statistic being 2.49 for AAA grade and 2.35 for BAA grade. Given the average level of variance risk premium of 18.30, that translates into an average effect on credit spread in the order of 9 to 15 basis points. Once the forecasting horizons increase to 3, 6, 9, and 12 months, the t-statistic for the variance difference variable become insignificant or marginal.

When we combine the variance risk premium with the short rate level together, both become more significant at the short horizons. For example, at one month horizon, the t-statistic for short rate is -4.59 for AAA and -3.53 for BAA; while for variance premium is 4.26 and 3.77 respectively. In fact, the variance risk premium variable maintains at least a marginal significance in the joint regressions even at the 12 month horizon, even though the short rate drops out as insignificant beyond the 6 month horizon. Judging from the R^2's, e.g, at the 1 month horizon, the univariate R^2 for variance risk premium is about 5-6 percent, but its contribution to the joint R^2 is about 8-10 percent, on top of what the short rate level has already accomplished—32 percent.

This is an important finding, in that the variance risk premium of implied-expected variance difference captures an important component in credit risk premium that is independent with the fundamental risk being captured by the short-term interest rate. We provide some

[11] If one includes the term spread alone, it is marginally significant with R^2 of 8-9 percent and t-statistics of 1.73 and 2.09. However, when short rate and term spread are combined together, term spread is driven out with t-statistics being -0.94 and -0.93. These tabular results are available upon request.

macroeconomic interpretation for such an effect in the next section.

4 A Model of Macroeconomic Uncertainty

It is challenging to provide a conceptual framework to reconcile the above findings on the short-run predictability on equity, bond, currency, and credit market from the variance risk premium variable. Here I rely on a self-contained general equilibrium model with stochastic consumption volatility-of-volatility, as explored by Tauchen (2005) and Bollerslev, Tauchen, and Zhou (2009), to give a fully rational *qualitative* interpretation of the short-term asset predictability puzzles. One should admit upfront that such a stylized model cannot provide a unified *quantitative* explanation various asset pricing puzzles, within a same parameter setting and being constrained by matching the consumption growth and volatility dynamics. In fact, most consumption-based asset pricing models cannot even qualitatively explain the existence of risk premia in bond, currency, and credit market; and can only explain a very small portion of the observed equity premium (Mehra and Prescott, 1985).

4.1 Uncertainty Dynamics and Price-Dividend Ratio

Suppose that the geometric growth rate of consumption in the economy, $g_{t+1} = \log(C_{t+1}/C_t)$, is unpredictable,

$$g_{t+1} = \mu_g + \sigma_{g,t} z_{g,t+1}, \tag{9}$$

where μ_g denotes the constant mean growth rate, $\sigma_{g,t}$ refers to the conditional volatility of the growth rate, and $\{z_{g,t}\}$ is an i.i.d. N(0, 1) innovation process. Further, assume that the economic uncertainty or the consumption volatility process is governed by the following discrete-time stochastic volatility process

$$\sigma^2_{g,t+1} = a_\sigma + \rho_\sigma \sigma^2_{g,t} + \sqrt{q_t} z_{\sigma,t+1}, \tag{10}$$

$$q_{t+1} = a_q + \rho_q q_t + \varphi_q \sqrt{q_t} z_{q,t+1}, \tag{11}$$

where the parameters satisfy $a_\sigma > 0, a_q > 0$, $|\rho_\sigma| < 1, |\rho_q| < 1$, $\varphi_q > 0$, and $\{z_{\sigma,t}\}$ and $\{z_{q,t}\}$ are i.i.d. N(0, 1) processes jointly independent of $\{z_{g,t}\}$. The stochastic volatility process $\sigma^2_{g,t+1}$ represents time-varying economic uncertainty in consumption growth, with the volatility-of-volatility process q_t in effect inducing an additional source of temporal variation in that uncertainty process. The time-variation in $\sigma^2_{g,t+1}$ alone is only the one of two components that drives the equity risk premium for the "consumption risk"; while the time-variation q_t is not only responsible for "uncertainty risk" in equity risk premium, but also constitutes the dominant source of bond, currency, and credit risk premia as explained bellow.

The representative agent in the economy is equipped with Epstein-Zin-Weil recursive preferences. Consequently, the logarithm of the intertemporal marginal rate of substitution (IMRS), $m_{t+1} \equiv \log(M_{t+1})$, may be expressed as,

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{t+1}, \tag{12}$$

and

$$\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}, \tag{13}$$

where δ denotes the subjective discount factor, ψ equals the intertemporal elasticity of substitution, γ refers to the coefficient of risk aversion, and r_{t+1} is the time t to $t+1$ return on the consumption asset. We will maintain the key assumptions that $\gamma > 1$, implying that the agents are more risk averse than the log utility investor; and $\psi > 1$, which in turn implies that $\theta < 0$ and that agents prefer an earlier resolution of economic uncertainty. These restrictions ensure, among other things, that uncertainty or volatility risk in asset markets carries a *positive* risk premia.

Let w_t denote the logarithm of the price-consumption or wealth-consumption ratio, of the asset that pays the consumption endowment, $\{C_{t+i}\}_{i=1}^{\infty}$. The standard solution method for finding the equilibrium in a model like the one defined above then consists in conjecturing a solution for w_t as an affine function of the state variables, $\sigma^2_{g,t}$ and q_t,

$$w_t = A_0 + A_\sigma \sigma^2_{g,t} + A_q q_t, \tag{14}$$

solving for the coefficients A_0, A_σ and A_q, using the standard Campbell-Shiller approximation $r_{t+1} = \kappa_0 + \kappa_1 w_{t+1} - w_t + g_{t+1}$. The resulting equilibrium solutions for the three coefficients may be expressed as,

$$A_0 = \frac{\log\delta + (1-\frac{1}{\psi})\mu_g + \kappa_0 + \kappa_1\left[A_\sigma a_\sigma + A_q a_q\right]}{(1-\kappa_1)}, \quad (15)$$

$$A_\sigma = \frac{(1-\gamma)^2}{2\theta(1-\kappa_1\rho_\sigma)}, \quad (16)$$

$$A_q = \frac{1-\kappa_1\rho_q - \sqrt{(1-\kappa_1\rho_q)^2 - \theta^2\kappa_1^4\varphi_q^2 A_\sigma^2}}{\theta\kappa_1^2\varphi_q^2}. \quad (17)$$

The aforementioned restrictions that $\gamma > 1$ and $\psi > 1$, hence $\theta < 0$, readily imply that the impact coefficient associated with both of the volatility state variables are negative; i.e., $A_\sigma < 0$ and $A_q < 0$.

So if consumption risk and uncertainty risk are high, the price-dividend ratio is low, hence risk premia are high. This would be the case if the intertemporal elasticity of substitution (IES) is bigger than one, when the intertemporal substitution effect dominates the wealth effect. In response to high economic uncertainty risks, agents sell more assets, and consequently the wealth-consumption ratio falls. In the standard power utility model, the restriction $\gamma > 1$ implies $\psi < 1$, hence $\theta > 0$. Then these impact coefficients would be positive $A_\sigma > 0$ and $A_q > 0$. Consequent the wealth effect dominates the substitution effect, which implies the counterintuitive result that when economic uncertainty risks are high, price-dividend ratio rises and risk premia are low.

4.2 Variance Risk Premium Dynamics

The conditional variance of the time t to $t+1$ return, $\sigma_{r,t}^2 \equiv \mathrm{Var}_t(r_{t+1})$, can be shown as

$$\sigma_{r,t}^2 = \sigma_{g,t}^2 + \kappa_1^2\left(A_\sigma^2 + A_q^2\varphi_q^2\right)q_t, \quad (18)$$

which is directly influenced by each of the two stochastic factors, the consumption volatility, $\sigma_{g,t}^2$, and the volatility of consumption volatility, q_t. As demonstrated bellow, the existence of time-varying volatility of volatility, q_t, is critical for generating the time-variation in variance risk premium; as well as for explaining the short-run time-variation in equity, bond, currency, and credit risk premia.

In connection with existing literature on option-implied volatility risk premium (Heston, 1993; Bakshi and Kapadia, 2003), one can define the variance risk premium as the difference between risk-neutral and objective expectations of the return variance,

$$
\begin{aligned}
VRP_t &\equiv \mathrm{E}_t^Q\left(\sigma_{r,t+1}^2\right) - \mathrm{E}_t^P\left(\sigma_{r,t+1}^2\right) \\
&\approx (\theta - 1)\kappa_1\left[A_\sigma + A_q\kappa_1^2\left(A_\sigma^2 + A_q^2\varphi_q^2\right)\varphi_q^2\right] q_t \ > 0 \ , \qquad (19)
\end{aligned}
$$

where the approximation comes from the fact that the model implied risk-neutral conditional expectation $E_t^Q(\sigma_{r,t+1}^2) \equiv E_t(\sigma_{r,t+1}^2 M_{t+1}) E_t(M_{t+1})^{-1}$ cannot computed in closed form, and a log-linear approximation is applied $E_t^Q(\sigma_{r,t+1}^2) \approx \log\left[e^{-r_{f,t}} E_t\left(e^{m_{t+1}+\sigma_{r,t+1}^2}\right)\right] - \frac{1}{2}\mathrm{Var}_t\left(\sigma_{r,t+1}^2\right)$, where $r_{f,t}$ is the one period risk-free interest rate.

One key observation here is that any temporal variation in the endogenously generated variance risk premium, is due solely to the volatility-of-volatility or economic uncertainty risk, q_t, but not the consumption growth risk, $\sigma_{g,t+1}^2$, which is also reflected later on in other risk premia. Moreover, provided that $\theta < 0$, $A_\sigma < 0$, and $A_q < 0$, as would be implied by the agents' preference of an earlier resolution of economic uncertainty (intertemporal elasticity of substitution—IES—bigger than one), this difference between the risk-neutral and objective expected variation is guaranteed to be positive. If $\varphi_q = 0$, and therefore $q_t = q$ is constant, the variance premium reduces to, $\mathrm{E}_t^Q(\sigma_{r,t+1}^2) - \mathrm{E}_t^P(\sigma_{r,t+1}^2) \ = \ (\theta - 1)\kappa_1 A_\sigma q$, which, of course, would also be constant. This reflects the classical result that in consumption-based asset pricing model, if consumption volatility is constant, then variance risk premium is zero. Even if consumption volatility is stochastic, as long as the volatility-of-volatility is constant, then there would be no time-variation or unconditional skewness and kurtosis in the observed variance risk premium.

Comparing the expression of variance risk premium in equation (19) to the expression for the conditional variance in equation (18), suggests that the variance risk premium should serve as an almost perfect measure of the elusive economic uncertainty risk, or q_t, as advocated by the modeling framework adopted here. This motivates the approach based on information from derivatives markets (or Q-measure information) for better estimating the so-far elusive notion of economic uncertainty risk.

To more directly gauge how the variance risk premium dynamics can be generated in the model proposed here, I use the calibration parameter setting in Bollerslev, Tauchen, and Zhou (2009). In particular, the values for $\delta = 0.997$, $\gamma = 10$, $\psi = 1.5$, $\mu_g = 0.0015$, and $\mathrm{E}(\sigma_g) = 0.0078^2$ are adapted from Bansal and Yaron (2004) to match the consumption dynamics. Additionally, one fixes $\kappa_1 = 0.9$, the persistence of the variance at $\rho_\sigma = 0.978$, the persistence of the volatility-of-volatility at $\rho_q = 0.80$, the expected volatility-of-volatility at $\mathrm{E}(q) = a_q(1-\rho_q)^{-1} = 1 \times 10.0^{-6}$, and the volatility parameter of that process at $\phi_q = 1 \times 10.0^{-3}$. The resulting equity risk premium is 7.79 percent and riskfree rate is 0.69 percent.

As shown in Table 8, the model-implied variance risk premium has a mean of 3.69 (percentage squared, not annualized) and a standard deviation of 6.47, which are much smaller than the observed values of 18.30 and 22.69. On the other hand, the model produces very realistic values in skewness of 2.82 and kurtosis of 14.92, which are very close to the observed values of 2.79 and 16.62. Finally the model-implied persistence coefficient is somewhat high at 0.80 as opposed to the low value of 0.26 reflected in the observed data. These results are non-trivial in that the model parameters are only calibrated to match the consumption dynamics and equity risk premiums, without targeting the variance risk premium in the first place. More importantly, the evidence shows that a stochastic volatility-of-volatility model can generate realistic skewness and kurtosis in variance risk premium, similar as those generate in Drechsler and Yaron (2008), with common jumps in consumption growth and consumption volatility.

4.3 Calibrating Equity Return Predictability

Given the solution of price-dividend ratio in this economy, one can solve for equity market return from time t to $t+1$, as shown in Tauchen (2005),

$$\begin{aligned} r_{t+1} \;=\; & -\log\delta \;+\; \frac{1}{\psi}\mu_g \;-\; \frac{(1-\gamma)^2}{2\theta}\sigma_{g,t}^2 \;+\; (\kappa_1\rho_q - 1)A_q q_t \;+ \\ & \sigma_{g,t} z_{g,t+1} \;+\; \kappa_1\sqrt{q_t}\,\left[A_\sigma z_{\sigma,t+1} + A_q \varphi_q z_{q,t+1}\right], \end{aligned} \tag{20}$$

Of particular interest is the model-implied equity risk premium,

$$\mathrm{E}_t\left(r_{t+1}\right) - r_{f,t} \;=\; \gamma\sigma_{g,t}^2 \;+\; (1-\theta)\kappa_1^2(A_q^2\varphi_q^2 + A_\sigma^2)q_t \;>\; 0. \tag{21}$$

The premium is composed of two separate terms. The first term, $\gamma\sigma_{g,t}^2$, is compensating the classic consumption risk term in consumption CAPM. The term doesn't really represent a genuine variance risk premium *per se*, however. Instead, it arises within the model as the presence of time-varying volatility in effect induces shifts in the market price of consumption risk. The second term, $(1-\theta)\kappa_1^2(A_q^2\varphi_q^2 + A_\sigma^2)q_t$, represents a true premium for volatility risk. It is a confounding of a risk premium on shocks to economic uncertainty, $z_{\sigma,t+1}$, and shocks to the volatility-of-uncertainty, $z_{q,t+1}$. As such it represents a fundamentally different source of risk from that of the traditional consumption risk term.

The existence of the volatility or uncertainty risk premium depends crucially on the dual assumptions of recursive utility, or $\theta \neq 1$, as uncertainty would otherwise not be a priced factor; and time varying volatility-of-uncertainty, in the form of the q_t process. This additional source of uncertainty is absent in the model of Bansal and Yaron (2004). The restrictions that $\gamma > 1$ and $\psi > 1$ implies that the variance risk premium embedded in the equity risk premium is always positive by construction, $(1-\theta)\kappa_1^2(A_q^2\varphi_q^2 + A_\sigma^2)q_t > 0$, as more risk requires more return. And since the variance risk premium embedded in equity returns loads on the same uncertainty risk factor in the variance risk premium, q_t, the latter becomes a prefect predictor for the short-run equity premium variation induced by the stochastic economic uncertainty component.

The calibration of equity return prediction here uses the exactly the same parameter calibration setting as in the last subsection. It is clear that the model-implied slope coefficients depicted in the top panel of Figure 5 declines monotonically with the return horizon. It starts out as 0.26 at one month horizon and declines to 0.10 at the annual horizon, which is only slightly lower than the estimated ones reported in Table 2—the one month slope coefficient is 0.43 and one year is 0.12, and the downward sloping pattern is very similar. Turning to the model-implied R^2's depicted in the bottom panel in the figure, the degree of predictability starts out fairly low at the monthly horizon, rising to its maximum around quarterly horizon, and gradually tapering off thereafter for longer return horizons. The model-implied maximum R^2 is around 3.5 percent at 4 month horizon, versus the empirical maximum around 5.86 percent at 2 month horizon.

As these calibrations make clear, the simple stylized general equilibrium model can give rise to quite sizable regression coefficients and return predictability. Importantly, the calibrations also reveal a general hump shape in the implied R^2 as a function of the return horizon. At an intuitive level, the variance risk premium on the right-hand-side of the predictive regression may be seen as a pure volatility bet where everything else gets "risk neutralized" out. Since volatility is explicitly priced under the Epstein-Zin-Weil recursive preference structure, this variance difference earns exactly that volatility risk premium and nothing else. The price of this risk changes if the variance of the priced factor changes. But this corresponds exactly to the volatility-of-volatility, or the q_t process within the theoretical model.

4.4 Explaining Bond Risk Premia

From the solution to the price-dividend ratio, it is straightforward to deduce the reduced form expressions for other variables of interest. In particular, the risk-free rate at time t must satisfy the following relation, as shown in Tauchen (2005),

$$\begin{aligned} r_{ft} \;=\; \theta \log \delta \;-\; \gamma \mu_g \;+\; (\theta - 1)\left[\kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1(A_\sigma a_{\sigma c} \;+\; A_q a_q)\right] \\ +(\theta - 1)\left[A_\sigma(\kappa_1 \rho_{\sigma g} - 1)\sigma_{g,t}^2 \;+\; A_q(\kappa_1 \rho_q - 1)q_t\right] \\ +\frac{1}{2}\gamma^2 \sigma_{g,t}^2 + \frac{1}{2}(\theta - 1)^2 \kappa_1^2 (A_\sigma^2 + A_q^2 \varphi_q^2) q_t \end{aligned} \tag{22}$$

where the last line are well known *Jensen's Inequality* terms, and second line are the genuine time-varying risk premia in real interest rates. The existence of the risk premia depends crucially on the dual assumptions of recursive utility, or $\theta \neq 1$, as otherwise only the *Jensen's Inequality* terms contribute to the time-variation in short rate; and time varying volatility of volatility, in the form of the q_t process, as otherwise the genuine risk premium in short rate would just be constants. The restrictions that $\gamma > 1$ and $\psi > 1$ implies that the risk premia are positive. It is instructive to reflect on the contrast with the standard consumption based asset pricing model, when the consumption volatility is constant. In that case, the risk-free rate is constant, $r_{ft} = \theta \log \delta - \gamma \mu_g + \frac{1}{2}\gamma^2 \sigma_g^2$; and there will be no "genuine" risk premium embedded in the short interest rate, except for the constant *Jensen's Inequality* term.

The equilibrium bond yield in this model is an affine function of the state variables,

$$y_t(n) = -\frac{1}{n} \left[\begin{array}{ccc} A(n) & B(n) & C(n) \end{array} \right] \left[\begin{array}{ccc} 1 & \sigma^2_{g,t} & q_t \end{array} \right]', \tag{23}$$

where the coefficients $A(n)$, $B(n)$, and $C(n)$ are shown in Zhou (2008). Let $rx_{t+1}(n-1)$ be the bond excess return from t to $t+1$ for an n-period bond, or the expected excess one month holding period return in Section 3, then the risk premia restriction becomes

$$rp^n_t = \left[B(n-1)(\theta-1)\kappa_1 A_\sigma + C(n-1)(\theta-1)\kappa_1 A_q \varphi^2_q\right] q_t \;>\; 0, \tag{24}$$

where the risk premia genuinely has two time-varying components—consumption risk and uncertainty risk, but they are co-linear in only one state variable q_t. This is driven by the fact that the variances of both volatility process and volatility-of-volatility process are loading on the same state variance, q_t. Please note that in the standard consumption-based asset pricing model, bond risk premium is zero by construction, because there is no time-varying volatility. The most interesting point is that, even if the consumption volatility is time-varying, as long as the volatility-of-volatility is constant, then bond risk premia must be constant. This result holds because we do not assume any statistical correlation between the volatility and consumption innovations.

Comparing the bond risk premia (24) with the variance risk premia (19), it is obvious that there two risk premia are perfectly co-linear. Therefore in this modeling framework, variance risk premia can perfectly predict the bond risk premia, which certainly contradicts the empirical result in Section 3. However, even if such a stylized model cannot quantitatively explains the bond return predictability from variance risk premium over various forecasting horizons, it does qualitatively explains the existence of bond risk premium variations within a fully rational model. This is a non-trivial task for many consumption-based asset pricing models, without imposing exogenous inflation dynamics.

The calibration parameter setting for variance and equity risk premiums in the last two subsection cannot qualitatively replicate the bond risk premia, because the high risk aversion and low consumption volatility would drive the long term real rate to be hugely negative. Three key parameters need to be changed to explain the bond risk premia: (1) risk aversion drops from $\gamma = 10$ to $\gamma = 2$, (2) unconditional consumption volatility increases from

$E(\sigma_g) = 0.0078^2$ to $E(\sigma_g) = 0.0025$, and (3) unconditional volatility-of-volatility increases from $E(q) = 1 \times 10.0^{-6}$ to $E(q) = 6 \times 10.0^{-6}$. A material implication of these parameter values is that the consumption volatility is increased from 2.7 percent to 17.3 percent, which may be justified if one "leverages" up the dividend growth several times larger than the consumption growth and removes the cointegration relationship between consumption growth and dividend growth (Abel, 1999). For example, the dividend volatility is levered up to 5.96 times of the consumption volatility in Bansal, Kiku, and Yaron (2007).

Such a calibration parameter setting produces an equity premium of 6.05 percent, riskfree rate 0.78 percent, and five-year real rate 0.48 percent. Figure (6) reports the empirically estimated and model implied bond risk premiums for the two to six month t-bills for holding one month. Since the bond risk premium inside the model is entirely driven by the uncertainty risk factor q_t, which is also fully loaded in the variance risk premium, the prediction R^2 should be one and hence is omitted in the figure. Empirically, the estimated bond risk premium increases from 11 to 24 basis points from two to six month maturity, with a convex shape at the short end. The calibrated model produces bond risk premium with a linear shape and increasing from 5 to 23 basis points. So the matching of bond risk premia at the long end is better than the short end.

One should be cautioned that such a result is achieved by levering up the dividend volatility relative to consumption volatility. Alternatively one can impose exogenous or endogenous inflation dynamics that the nominal term premium and its higher order moments may be matched without levering up the dividend volatility. I leave this for future research.

4.5 Extension to Forward Premium

To provide some qualitative justification for the short-run forecastability in currency risk premium from the variance risk premium variable, one need to extend to a two-country framework as in Bansal and Shaliastovich (2008b). Following the equilibrium no-arbitrage argument in Backus, Foresi, and Telmer (2001) and assuming that the corresponding foreign country has the same model specification as the home country (except that all the variables in foreign countries are labeled with a star); then the expected depreciation of the domestic currency for one period ahead can be shown as

$$\mathrm{E}_t\left(s_{t+t}\right)-s_t=A+B\left(\sigma_{g,t}^2-\sigma_{g,t}^{2*}\right)+C\left(q_t-q_t^*\right) \tag{25}$$

where s_t is the log exchange rate—domestic currency over foreign currency, $\sigma_{g,t}^{2*}$ and q_t^* are the foreign consumption risk and uncertainty risk variables, and A, B, C are coefficients of the underlying structural parameters (Zhou, 2008). Note that by construction $B>0$ and $C>0$, which means a positive shock to domestic consumption volatility or volatility-of-volatility in equilibrium, causes the dollar price of the foreign currency to fall immediately, hence an *expected* appreciation of the foreign currency.

As such the result in equation (25) could potentially explain *qualitatively* the positive slope coefficient in regressing the exchange rate on the US variance risk premium in Table 6, with the variance risk premium *positively* loads on the US uncertainty risk factor q_t. This would be the case for the Euro and Pound, if the US uncertainty risk factor q_t dominates its foreign counterpart q_t^*—more volatile and persistent. On the other hand, the near zero and insignificant slope coefficient found in regressing the Yen currency on the US variance risk premium in Table 6, could potentially be explained by the dominance of the Japan uncertainty risk factor q_t^* over the US uncertainty risk factor q_t in terms of variability and persistence. A thorough empirical testing of such a model implication requires constructions of the variance risk premium proxies in the European, British, and Japanese economies. Given the evidence that consumption growth risk does explain the long-run variation in currency risk premia (Lustig and Verdelhan, 2007), the extension to a richer consumption volatility risk as examined here may help to explain the short-run variation in currency risk premia.

4.6 Extension to Credit Spread

It is also challenging to incorporate the default risk of a representative firm into the current modeling framework. The strategy could follow Chen (2008) and Bhamra, Kuehn, and Strebulaev (2009), where the recursive preference plus macroeconomic uncertainty generate richer dynamics in the credit spread dynamics. To fix the idea, assume in a Merton (1974) type model as in Chen, Collin-Dufresne, and Goldstein (2008), and the credit spread CS of a discount bond for a defaultable firm with T maturity can be shown as

$$\mathrm{CS}_t(T) = -\frac{1}{T}\log\left\{1 - \mathrm{LGD} \times \mathrm{Normal}\left[\mathrm{Normal}^{-1}(\mathrm{PD}) + \lambda\sigma\sqrt{T}\right]\right\}, \tag{26}$$

where LGD is the loss given default, PD is the real default probability, λ is the market price of asset risk, and σ is the asset return volatility. All these important variables are constants or deterministic in the original Merton model.

It is well known that such a simplified model cannot explain the high credit spread level and its time variation (Huang and Huang, 2003). Many equilibrium structural approaches (see, e.g., Chen, 2008; Bhamra, Kuehn, and Strebulaev, 2009, among others) can be viewed as letting the real default probability PD_t to be time-varying and countercyclical, with possible business cycle fluctuations of the firm's refinancing decision or default barrier. It is also possible to model the recovery rate or LGD_t as a stochastic time-varying process to help explain the credit spread puzzles. Chen, Collin-Dufresne, and Goldstein (2008) take a novel approach to allow the market price of risk λ_t to be driven by a countercyclical risk aversion factor in a habit persistence model (Campbell and Cochrane, 1999).

However, their conclusion that the long-run risk (LRR) model cannot adequately explain the credit spread puzzle is not a surprise; because by construction the original Bansal and Yaron (2004) model only has scholastic volatility but not stochastic volatility-of-volatility. As shown above, the time variation in the economic uncertainty risk or q_t is a dominant driving force for the bond and currency risk premiums, and presumably also holds true for the credit risk premium. This is equivalent to allow the asset return volatility σ_t to be countercyclical and depending on both the consumption risk and the uncertainty risk. Such an extension may be supported by the preliminary evidence in Zhang, Zhou, and Zhu (2009) that stochastic asset volatility can help structural models to explain the credit spread puzzles.

5 Conclusion

The implied-expected variance difference can be viewed as a measure for the variance risk premium. This paper provides consistent empirical evidence that the variance risk premium can significantly predict short-run equity returns, bond returns, forward premiums, and credit spreads. The documented return predictability peaks around one month and decline with the forecasting horizon. Importantly, such a short-term forecastability of risk premia is complementary to the established predictor—P/E ratio, forward rate, interest rate differential, and short rate level. This constitutes an important evidence that risk premia across major financial markets co-vary in short-term, and such a comovement seems to be driven by a common variance risk factor, measured by the implied-expected variance difference.

Such a common *short-run* risk factor may be a proxy for the macroeconomic uncertainty or consumption volatility risk that varies independently with the consumption growth risk—the main focus of *long-run* risk models (Bansal and Yaron, 2004). The empirical results are consistent with a general equilibrium model incorporating the effects of such a time-varying economic uncertainty component, where the uncertainty risk is priced only under the recursive preference. The paper provides calibration evidence that the short-run predictability of equity and bond markets can be qualitatively replicated by the model, although the extensions to currency and credit markets require additional variable construction and model sophistication.

Although the stylized model examined here can provide qualitative justification of the short-run predictability of major asset market returns from the variance risk premium variable, it is not rich enough to *simultaneously* explain such effects within a same parameter setting. More importantly, to jointly interpret the *long-run* and *short-run* risk factors in these markets, additional model sophistication is needed to *quantitatively* replicate various predictability puzzles established in the literature. In addition, the short-run forecastability of variance risk premium documented here as in the time-series domain need to be reconciled with the cross-sectional evidence of asset market returns. We leave these challenging issues for future research.

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Table 1 Summary statistics

The sample period extends from January 1990 to December 2008, except for exchange rates in Panel D starting from January 1999 when Euro is created. All variables are reported in annualized percentage form whenever appropriate. Panel A reports the variance risk premiums using different methods for estimating the physical expectation of the realized variance: (1) ex post realized variance, (2) lagged realized variance, (3) 12 month moving average, (4) recursive AR(12) forecast, and (5) full sample AR(12) forecast. The series of implied variance (end-of-the-month observation) and realized variance (summation over the month) are available from `http://sites.google.com/site/haozhouspersonalhomepage/`. Panel B reports S&P500 market returns, log price-earning ratio, with the variance risk premium. Panel C reports the Treasury zero coupon bond with 1-6 month maturities. To conserve space, only the 1-month holding period returns for 3- and 6-month t-bills and their matching 2- and 5-month forward rate are reported. Panel D report the log exchange rate of Euro, Pound, and Yen against US Dollar; with their matching interest rate differentials. Panel E reports the Moody's AAA and BAA credit spread indices with the US short rate level.

Panel A: Comparison of VRP_t with different RV_t forecasts

	RV_t	RV_{t-1}	$MA(12)$	Recursive $AR(12)$	Full Sample $AR(12)$
	Summary Statistics				
Mean	17.22	17.07	20.49	21.81	18.30
Std Dev	20.27	19.99	25.03	27.86	22.69
Skewness	-3.12	-3.32	4.19	4.39	2.79
Kurtosis	44.20	46.41	27.51	34.15	16.62
AR(1)	0.28	0.29	0.66	0.76	0.26
	Correlation Matrix				
RV_t	1.00	0.28	0.11	0.23	0.04
RV_{t-1}		1.00	0.06	0.12	0.18
$MA(12)$			1.00	0.63	0.75
Recursive $AR(12)$				1.00	0.21
Full Sample $AR(12)$					1.00

Panel B: Equity

	$r_t - r_{f,t}$	VRP_t	$\log(P_t/E_t)$
	Summary Statistics		
Mean	4.65	18.30	3.12
Std Dev	47.52	22.69	0.25
Skewness	-0.64	2.79	0.48
Kurtosis	4.27	16.62	2.53
AR(1)	-0.03	0.26	0.97
	Correlation Matrix		
$r_t - r_{f,t}$	1.00	0.17	-0.14
VRP_t		1.00	0.07
$\log(P_t/E_t)$			1.00

Panel C: Treasury Bill

	$xhpr^3_{t+1}$	$xhpr^6_{t+1}$	VRP_t	$f_t(2,1)$	$f_t(5,1)$
	Summary Statistics				
Mean	4.26	4.55	18.30	4.13	4.22
Std Dev	1.92	2.24	22.69	1.88	1.83
Skewness	-0.10	-0.22	2.79	-0.15	-0.24
Kurtosis	2.45	3.91	16.62	2.45	2.42
AR(1)	0.90	0.72	0.26	0.97	0.99
	Correlation Matrix				
$xhpr^3_{t+1}$	1.00	0.92	0.13	0.95	0.86
$xhpr^6_{t+1}$		1.00	0.18	0.80	0.81
VRP_t			1.00	0.07	0.11
$f_t(2,1)$				1.00	0.90
$f_t(5,1)$					1.00

Panel D: Foreign Exchange

	$\Delta Euro$	ΔGBP	ΔJPY	VRP_t	ΔIR_{Euro}	ΔIR_{GBP}	ΔIR_{JPY}
	Summary Statistics						
Mean	-1.55	1.06	-2.74	18.30	-0.07	1.54	-2.96
Std Dev	30.59	26.63	60.00	22.69	1.43	1.22	1.68
Skewness	-0.03	0.71	-0.21	2.79	-0.01	0.04	-0.03
Kurtosis	3.01	5.38	2.30	16.62	1.54	1.65	1.67
AR(1)	0.33	0.33	-0.44	0.26	0.98	1.00	1.01
	Correlation Matrix						
$\Delta Euro$	1.00	0.75	0.17	0.13	-0.15	-0.15	-0.15
ΔGBP		1.00	0.10	0.04	0.03	-0.02	0.05
ΔJPY			1.00	0.22	0.11	0.04	-0.01
VRP_t				1.00	0.11	0.07	0.05
ΔIR_{Euro}					1.00	0.86	0.86
ΔIR_{GBP}						1.00	0.94
ΔIR_{JPY}							1.00

Panel E: Credit Spread

	AAA	BAA	VRP_t	$r_{f,t}$
	Summary Statistics			
Mean	1.25	2.14	18.30	4.24
Std Dev	0.45	0.68	22.69	1.83
Skewness	0.89	2.15	2.79	-0.20
Kurtosis	3.18	10.50	16.62	2.37
AR(1)	0.98	1.05	0.26	0.99
	Correlation Matrix			
AAA	1.00	0.90	0.15	0.30
BAA		1.00	0.14	0.32
VRP_t			1.00	-0.09
$r_{f,t}$				1.00

Table 2 Equity Returns, Variance Risk Premia, and P/E ratios

The sample period extends from January 1990 to December 2008. All of the regressions are based on monthly observations. Regression takes the form

$$xr_{t+h} \ = \ b_0(h) \ + \ b_1(h)\ VRP_t \ + \ b_2(h)\ \log\left(P_t/E_t\right) \ + \ u_{t+h,t},$$

where the excess return horizon h goes out to 24 months. Robust t-statistics following Newey and West (1987) are reported in parentheses. All variable definitions are identical to Table 1 Panel B.

Monthly Horizon	1	3	6	9	12	15	18	24
Constant	-2.92	-2.02	0.79	2.09	3.02	2.88	3.63	4.12
	(-0.54)	(-0.43)	(0.20)	(0.53)	(0.81)	(0.76)	(0.97)	(1.07)
VRP_t	0.43	0.35	0.21	0.16	0.12	0.13	0.08	0.05
	(3.28)	(2.98)	(2.40)	(2.64)	(3.00)	(3.16)	(1.95)	(1.20)
Adj. R^2 (%)	2.55	5.86	4.55	3.37	2.21	2.95	1.18	0.16

Monthly Horizon	1	3	6	9	12	15	18	24
Constant	82.97	76.90	69.45	68.40	70.27	67.13	64.71	69.52
	(2.08)	(1.92)	(1.85)	(2.08)	(2.43)	(2.45)	(2.31)	(2.26)
$\log(P_t/E_t)$	-25.03	-23.22	-20.71	-20.26	-20.75	-19.73	-18.94	-20.45
	(-1.91)	(-1.77)	(-1.67)	(-1.85)	(-2.14)	(-2.16)	(-2.05)	(-2.04)
Adj. R^2 (%)	1.38	4.42	7.73	10.76	13.93	14.27	15.03	20.22

Monthly Horizon	1	3	6	9	12	15	18	24
Constant	83.89	77.67	70.20	69.16	70.69	67.31	64.62	69.19
	(1.88)	(1.80)	(1.77)	(1.99)	(2.33)	(2.31)	(2.21)	(2.18)
$\log(P_t/E_t)$	-27.93	-25.64	-22.31	-21.54	-21.70	-20.62	-19.49	-20.72
	(-1.85)	(-1.80)	(-1.70)	(-1.85)	(-2.13)	(-2.12)	(-2.02)	(-2.00)
VRP_t	0.46	0.38	0.24	0.18	0.14	0.15	0.10	0.06
	(2.88)	(2.88)	(2.55)	(2.53)	(3.06)	(3.03)	(2.13)	(1.42)
Adj. R^2 (%)	4.39	11.35	13.59	15.55	17.46	18.58	17.12	20.95

Table 3 Bond Returns and Variance Risk Premia

The sample period extends from January 1990 to December 2008. All of the regressions are based on monthly observations. Regression takes the form

$$xhpr^n_{t+h} \;=\; b^n_0(h) \;+\; b^n_1(h)\, VRP_t \;+\; u^n_{t+h,t},$$

where $h = 1, 2, 3, 4, 5$ month and $n = 2, 3, 4, 5, 6$ month. Robust t-statistics following Newey and West (1987) are reported in parentheses. All variable definitions are identical to Table 1 Panel C.

Holding Period	2 Month Bill	3 Month Bill	4 Month Bill	5 Month Bill	6 Month Bill
1 Month/Const.	0.33	0.47	0.43	0.60	0.62
	(4.00)	(5.66)	(4.67)	(5.35)	(4.34)
VRP_t	5.96e-3	5.55e-3	6.88e-3	9.85e-3	13.12e-3
	(2.38)	(2.33)	(2.86)	(2.76)	(3.23)
Adj. R^2	4.20	2.77	2.86	4.24	4.57
2 Months/Const.		0.25	0.30	0.37	0.46
		(5.69)	(5.69)	(5.23)	(4.53)
VRP_t		1.76e-3	2.58e-3	4.26e-3	7.17e-3
		(1.22)	(2.05)	(1.95)	(2.03)
Adj. R^2		0.86	1.34	2.23	3.54
3 Months/Const.			0.16	0.26	0.32
			(4.97)	(5.39)	(4.23)
VRP_t			1.28e-3	1.98e-3	3.70e-3
			(1.19)	(1.24)	(1.42)
Adj. R^2			0.70	0.93	1.59
4 Months/Const.				0.18	0.28
				(5.74)	(4.74)
VRP_t				0.57e-3	0.84e-3
				(0.50)	(0.50)
Adj. R^2				-0.17	-0.22
5 Months/Const.					0.16
					(4.31)
VRP_t					0.44e-3
					(0.38)
Adj. R^2					-0.28

Table 4 Bond Returns and Lag Forward Rates

The sample period extends from January 1990 to December 2008. All of the regressions are based on monthly observations. Regression takes the form

$$xhpr^n_{t+h} \;=\; b^n_0(h) \;+\; b^n_2(h)\;[f_{t-1}(n-h,h) - y_{t-1}(h)] \;+\; u^n_{t+h,t},$$

where $h = 1, 2, 3, 4, 5$ month and $n = 2, 3, 4, 5, 6$ month. Robust t-statistics following Newey and West (1987) are reported in parentheses. All variable definitions are identical to Table 1 Panel C.

Holding Period	2 Month Bill	3 Month Bill	4 Month Bill	5 Month Bill	6 Month Bill
1 Month/Const.	0.02	0.03	0.02	0.04	0.05
	(4.58)	(5.68)	(4.26)	(4.09)	(4.59)
Forward Spread	0.49	0.41	0.48	0.37	0.33
	(7.99)	(8.88)	(6.32)	(4.12)	(3.80)
Adj. R^2	23.62	36.08	24.91	11.59	7.95
2 Months/Const.		0.04	0.04	0.04	0.07
		(5.53)	(5.93)	(3.83)	(4.13)
Forward Spread		0.24	0.30	0.43	0.28
		(3.16)	(4.57)	(4.87)	(2.56)
Adj. R^2		5.58	10.45	18.17	4.14
3 Months/Const.			0.03	0.05	0.06
			(4.84)	(4.53)	(3.76)
Forward Spread			0.28	0.33	0.35
			(4.15)	(5.24)	(3.21)
Adj. R^2			9.03	11.63	7.71
4 Months/Const.				0.05	0.07
				(5.36)	(5.34)
Forward Spread				0.20	0.25
				(3.21)	(5.46)
Adj. R^2				5.26	6.80
5 Months/Const.					0.05
					(5.09)
Forward Spread					0.15
					(2.65)
Adj. R^2					2.65

Table 5 Bond Returns, Variance Risk Premia, and Lag Forward Rates

The sample period extends from January 1990 to December 2008. All of the regressions are based on monthly observations. Regression takes the form

$$xhpr^n_{t+h} \;=\; b^n_0(h) \;+\; b^n_1(h)\; VRP_t \;+\; b^n_2(h)\; [f_{t-1}(n-h,h) - y_{t-1}(h)] \;+\; u^n_{t+h,t},$$

where $h = 1,2,3,4,5$ month and $n = 2,3,4,5,6$ month. Robust t-statistics following Newey and West (1987) are reported in parentheses. All variable definitions are identical to Table 1 Panel C.

Holding Period	2 Month Bill	3 Month Bill	4 Month Bill	5 Month Bill	6 Month Bill
1 Month/Const.	0.18	0.27	0.26	0.34	0.41
	(3.62)	(4.51)	(3.17)	(3.37)	(3.13)
Forward Spread	0.44	0.39	0.42	0.40	0.34
	(7.08)	(7.99)	(5.29)	(4.79)	(4.06)
VRP_t	3.65e-3	5.04e-3	4.85e-3	9.23e-3	11.10e-3
	(2.07)	(3.02)	(2.75)	(2.99)	(3.29)
Adj. R^2	22.98	33.23	19.25	15.47	11.18
2 Months/Const.		0.20	0.22	0.22	0.29
		(4.86)	(4.52)	(3.35)	(3.08)
Forward Spread		0.23	0.28	0.46	0.40
		(3.46)	(4.84)	(5.67)	(3.34)
VRP_t		1.25e-3	2.00e-3	3.11e-3	5.82e-3
		(0.99)	(1.92)	(1.89)	(2.11)
Adj. R^2		5.82	9.80	19.11	10.86
3 Months/Const.			0.13	0.18	0.21
			(4.31)	(4.07)	(2.86)
Forward Spread			0.22	0.35	0.42
			(3.24)	(5.98)	(4.03)
VRP_t			0.58e-3	1.37e-3	2.85e-3
			(0.67)	(1.08)	(1.28)
Adj. R^2			6.51	12.27	10.55
4 Months/Const.				0.15	0.22
				(4.85)	(3.91)
Forward Spread				0.18	0.24
				(3.38)	(5.72)
VRP_t				0.18e-3	0.52e-3
				(0.18)	(0.33)
Adj. R^2				4.13	5.25
5 Months/Const.					0.14
					(3.88)
Forward Spread					0.12
					(2.45)
VRP_t					0.15e-3
					(0.15)
Adj. R^2					1.64

Table 6 Exchange Rates, Variance Risk Premia, and Interest Rate Differentials

The sample period extends from January 1999 to December 2008. All of the regressions are based on monthly observations. Regression takes the form

$$s_{t+h} - s_t \;=\; b_0(h) \;+\; b_1(h)\, VRP_t \;+\; b_2(h)\; [y_t(h) \;-\; y_t^*(h)] \;+\; u_{t+h,t},$$

where s_t is the log exchange rate—domestic currency over foreign currency, while $y_t(h)$ and $y_t^*(h)$ are h-period zero coupon bond yields home and abroad. Robust t-statistics following Newey and West (1987) are reported in parentheses. All variable definitions are identical to Table 1 Panel D.

Euro	Const.	VRP_t	Adj. R^2	Const.	$y_t - y_t^*$	Adj. R^2	Const.	VRP_t	$y_t - y_t^*$	Adj. R^2
1 M	-0.68	3.35	9.48	-0.15	-0.26	1.31	-0.74	3.36	-0.33	12.08
	(-2.74)	(3.73)		(-0.55)	(-1.16)		(-3.46)	(4.59)	(-1.62)	
3 M	-0.62	1.31	-0.60	-0.47	-0.55	1.47	-0.75	1.74	-0.58	1.08
	(-0.82)	(0.57)		(-0.56)	(-0.74)		(-0.92)	(0.79)	(-0.79)	
6 M	-1.39	0.32	-0.89	-1.59	-1.49	7.82	-1.83	1.55	-1.51	7.14
	(-1.01)	(0.06)		(-1.17)	(-1.21)		(-1.25)	(0.34)	(-1.22)	
9 M	-2.87	2.67	-0.61	-3.10	-2.80	18.98	-3.97	5.52	-2.89	19.52
	(-1.64)	(0.35)		(-1.84)	(-1.86)		(-2.36)	(0.90)	(-1.98)	
12 M	-3.77	1.48	-0.88	-4.70	-4.05	26.15	-5.46	4.95	-4.12	26.13
	(-1.63)	(0.15)		(-2.26)	(-2.28)		(-2.22)	(0.61)	(-2.36)	
GBP	**Const.**	**VRP_t**	**Adj. R^2**	**Const.**	**$y_t - y_t^*$**	**Adj. R^2**	**Const.**	**VRP_t**	**$y_t - y_t^*$**	**Adj. R^2**
1 M	-0.41	3.03	10.34	-0.00	-0.04	-0.80	-0.45	2.13	0.07	3.25
	(-1.92)	(3.14)		(-0.01)	(-0.20)		(-1.26)	(3.11)	(0.30)	
3 M	-0.76	2.45	0.73	-0.07	-0.21	-0.35	-0.44	0.22	-0.22	0.41
	(-1.28)	(1.59)		(-0.07)	(-0.43)		(-0.45)	(1.63)	(1.63)	
6 M	-1.86	0.53	3.59	-0.15	-0.55	0.78	-1.04	0.54	-0.59	4.63
	(-1.61)	(1.87)		(-0.08)	(-0.59)		(-0.57)	(1.95)	(-0.66)	
9 M	-2.61	0.62	2.72	-0.10	-1.05	2.83	-1.14	0.62	-1.06	5.85
	(-1.57)	(1.38)		(-0.04)	(-0.78)		(-0.45)	(1.45)	(-0.81)	
12 M	-2.84	0.35	-0.08	-0.09	-1.58	5.17	-0.67	0.35	-1.58	5.15
	(-1.41)	(0.72)		(-0.03)	(-0.90)		(-0.20)	(0.76)	(-0.90)	
JPY	**Const.**	**VRP_t**	**Adj. R^2**	**Const.**	**$y_t - y_t^*$**	**Adj. R^2**	**Const.**	**VRP_t**	**$y_t - y_t^*$**	**Adj. R^2**
1 M	-0.20	-0.02	-0.85	-0.98	-0.26	-0.10	-1.00	0.01	-0.26	-0.96
	(-0.60)	(-0.02)		(-2.02)	(-1.67)		(-1.54)	(0.06)	(-1.62)	
3 M	0.28	-0.42	1.42	-2.38	-0.63	1.79	-1.61	-0.40	-0.61	3.06
	(0.41)	(-1.44)		(-2.76)	(-1.91)		(-1.71)	(-1.44)	(-1.88)	
6 M	-0.37	-0.22	-0.55	-4.25	-1.13	5.33	-3.86	-0.24	-1.14	4.89
	(-0.26)	(-0.93)		(-2.77)	(-1.79)		(-2.35)	(-1.26)	(-1.82)	
9 M	-0.04	-0.56	0.73	-6.58	-1.77	9.70	-5.61	-0.57	-1.77	10.57
	(-0.02)	(-1.49)		(-2.73)	(-1.99)		(-2.08)	(-1.68)	(-2.02)	
12 M	-0.52	-0.30	-0.53	-8.02	-2.19	13.62	-7.48	-0.34	-2.20	13.34
	(-0.20)	(-0.71)		(-2.20)	(-1.82)		(-1.89)	(-0.86)	(-1.85)	

Table 7 Credit Spreads, Variance Risk Premia, and Interest Rates

The sample period extends from January 1999 to December 2008. All of the regressions are based on monthly observations. Regression takes the form

$$\text{CS}_{t+h} \;=\; b_0(h) \;+\; b_1(h)\, VRP_t \;+\; b_2(h)\, r_{f,t} \;+\; u_{t+h,t},$$

where the credit spread of h month ahead is being forecasted. Robust t-statistics following Newey and West (1987) are reported in parentheses. All variable definitions are identical to Table 1 Panel E.

	Moody's AAA Bond Yield Spread				Moody's BAA Bond Yield Spread			
Horizon	Constant	$r_{f,t}$	VRP_t	Adj. R^2	Constant	$r_{f,t}$	VRP_t	Adj. R^2
1 Month	1.86	-0.14		32.68	3.06	-0.22		32.15
	(10.56)	(-3.94)			(8.18)	(-2.94)		
	1.17		4.68e-3	5.19	2.00		7.99e-3	6.64
	(11.19)		(2.49)		(14.21)		(2.35)	
	1.78	-0.15	5.85e-3	41.13	2.93	-0.23	9.76e-3	42.37
	(11.74)	(-4.59)	(4.26)		(9.10)	(-3.53)	(3.77)	
3 Months	1.77	-0.12		22.33	2.92	-0.18		21.83
	(9.33)	(-3.11)			(7.66)	(-2.52)		
	1.19		3.79e-3	2.61	2.01		7.37e-3	4.52
	(10.63)		(1.84)		(13.56)		(2.08)	
	1.71	-0.13	5.24e-3	27.75	2.81	-0.19	9.58e-3	29.77
	(10.00)	(-3.49)	(3.09)		(8.39)	(-3.00)	(2.87)	
6 Months	1.66	-0.09		12.78	2.75	-0.14		12.32
	(7.80)	(-2.17)			(6.92)	(-1.93)		
	1.21		2.72e-3	0.93	2.08		3.93e-3	0.77
	(10.37)		(1.19)		(12.31)		(1.35)	
	1.61	-0.10	4.26e-3	15.71	2.69	-0.15	6.25e-3	14.95
	(7.94)	(-2.43)	(2.17)		(7.08)	(-2.21)	(2.28)	
9 Months	1.53	-0.06		4.97	2.55	-0.09		5.00
	(6.65)	(-1.30)			(6.59)	(-1.35)		
	1.22		2.94e-3	1.17	2.07		5.15	1.64
	(9.89)		(1.29)		(12.36)		(1.48)	
	1.49	-0.07	3.90e-3	7.35	2.49	-0.10	6.67e-3	8.00
	(6.71)	(-1.49)	(1.86)		(6.92)	(-1.60)	(1.85)	
12 Months	1.37	-0.02		0.25	2.30	-0.03		0.23
	(5.82)	(-0.45)			(6.92)	(-0.57)		
	1.20		4.20e-3	2.85	2.06		5.49e-3	1.88
	(9.66)		(2.05)		11.49		(1.92)	
	1.33	-0.03	4.67e-3	3.78	2.24	-0.04	6.19e-3	2.67
	(5.88)	(-0.66)	(2.50)		(7.22)	(-0.80)	(2.10)	

Table 8 Sample Observed and Model-Implied Variance Risk Premium

The observed variance risk premium is based on the full sample AR(12) from Panel E of Table 1. The model calibration adopts the parameter settings in Bollerslev, Tauchen, and Zhou (2009) with $\delta = 0.997$, $\gamma = 10$, $\psi = 1.5$, $\mu_g = 0.0015$, and $E(\sigma_g) = 0.0078^2$, which are adapted directly from Bansal and Yaron (2004) to match consumption dynamics. Additionally, $\kappa_1 = 0.9$, $\rho_\sigma = 0.978$, $\rho_q = 0.80$, $a_q(1 - \rho_q)^{-1} = 1 \times 10.0^{-6}$, $\phi_q = 1 \times 10.0^{-3}$; which produces an equity premium 7.79 percent and a risk-free rate 0.69 percent.

	Full Sample AR(12)	Model-Implied
Mean	18.30	3.69
Std Dev	22.69	6.47
Skewness	2.79	2.82
Kurtosis	16.62	14.92
AR(1)	0.26	0.80

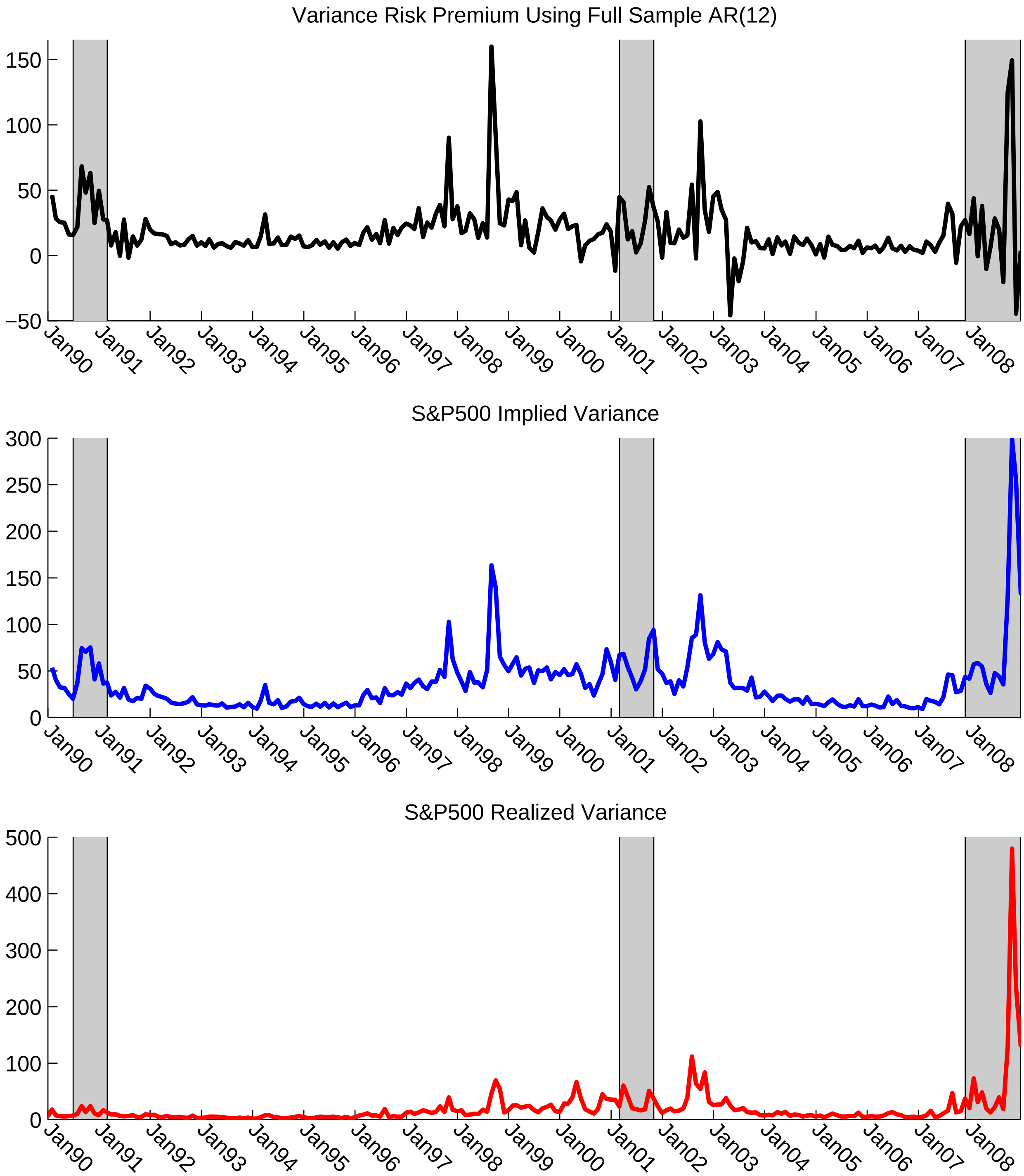

Figure 1 Variance Risk Premium, Implied and Realized Variances

This figure plots the variance risk premium or implied-expected variance difference (top panel), the implied variance (middle panel), and the realized variance (bottom panel) for the S&P500 market index from January 1990 to December 2008. The variance risk premium is based on the realized variance forecast from a full sample AR(12). The shaded areas represent NBER recessions.

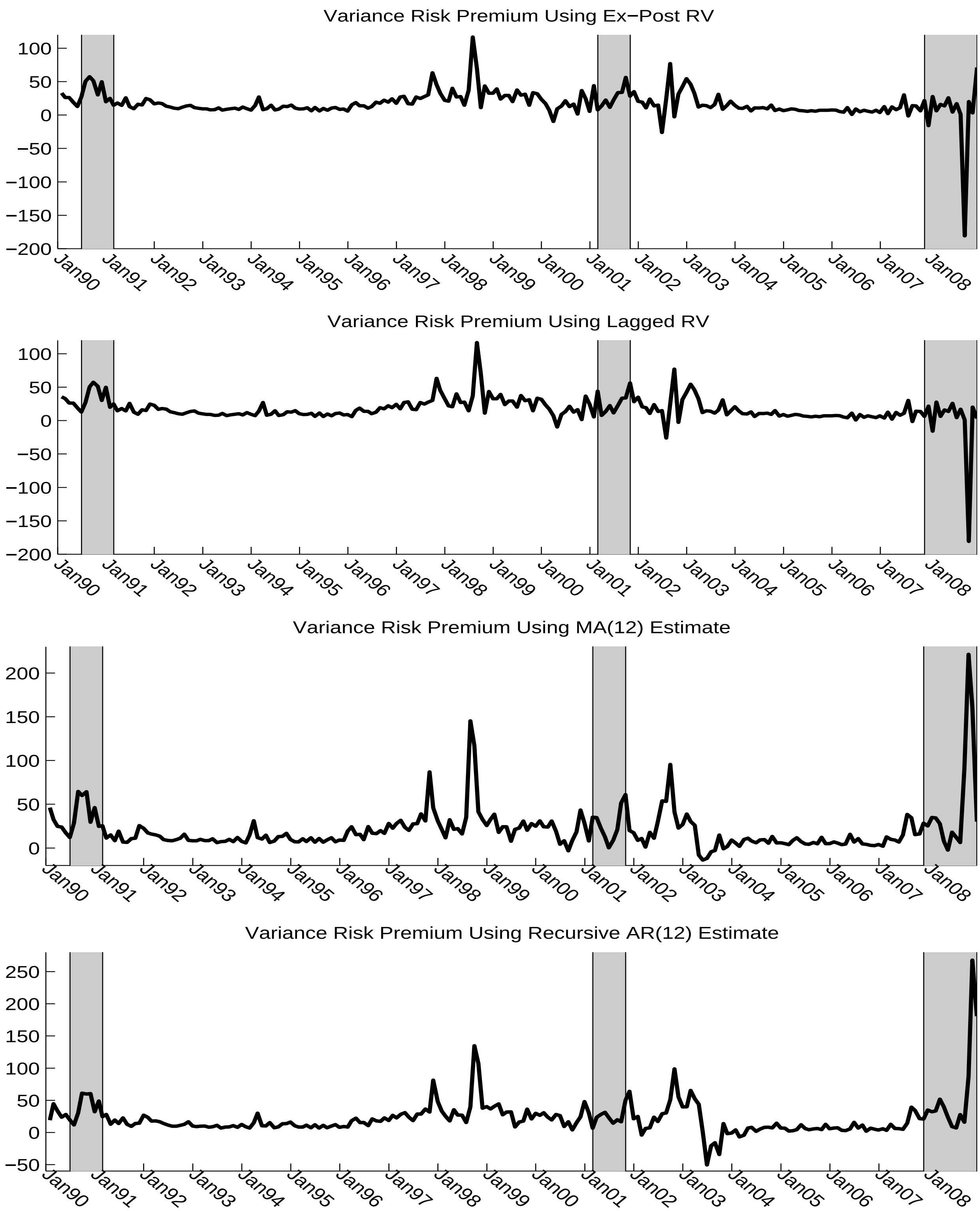

Figure 2 Variance Risk Premiums with Alternative RV_t Forecasts

The figure plots the variance risk premium series constructed using alternative ways to forecast the realized variance: (1) ex post RV_t, (2) lagged RV_{t-1}, (3) MA(12) estimate, and (4) recursive AR(12) estimate. The shaded areas represent NBER recessions.

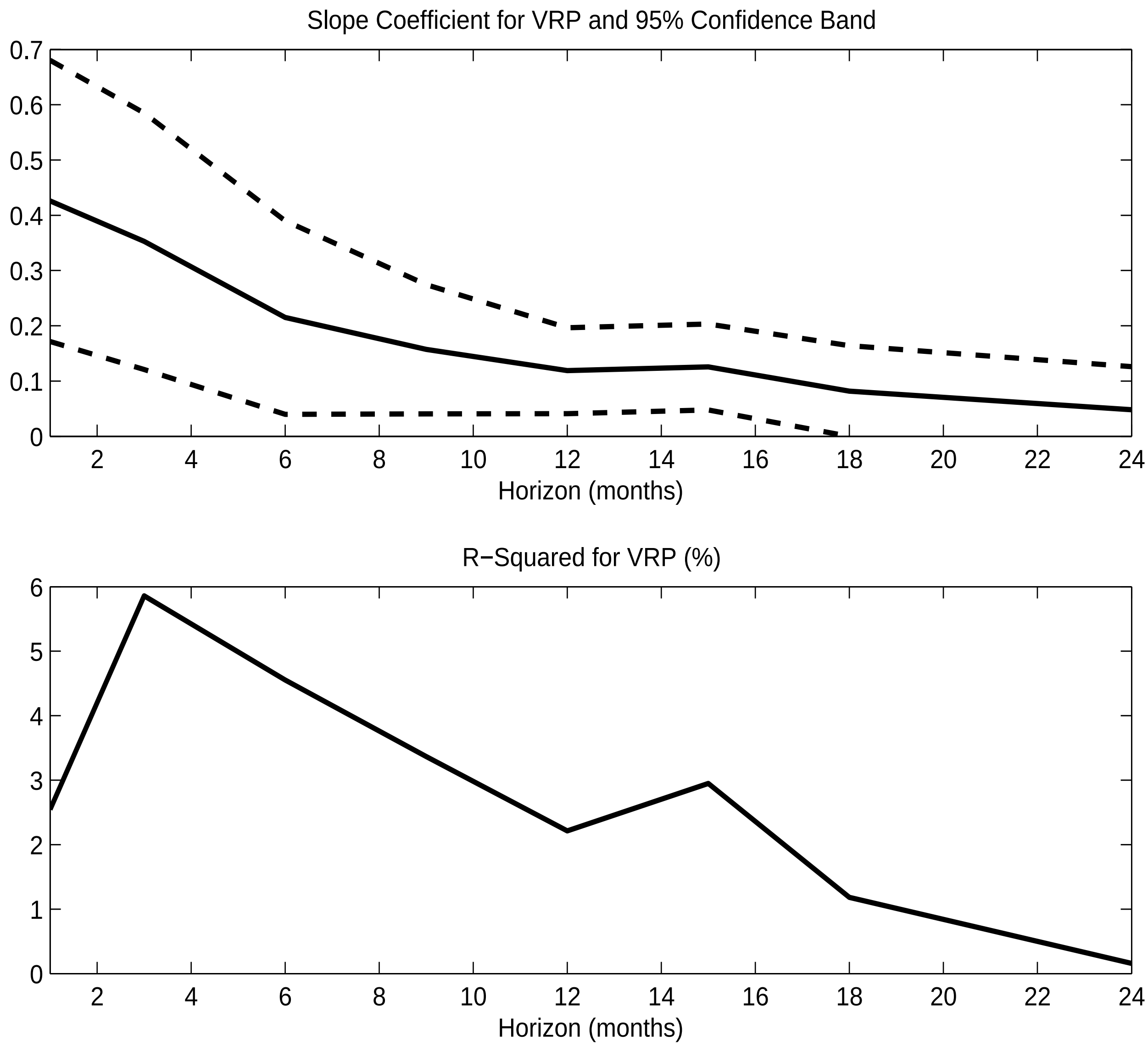

Figure 3 Estimated Slopes and R^2's of Equity Returns on VRP_t

The figure shows the estimated slope coefficients and pointwise 95 percent confidence intervals, along with the corresponding R^2's from the regressions of the scaled h-period S&P500 excess returns on the variance risk premium variable. All of the regressions are based on monthly observations from January 1990 to December 2008.

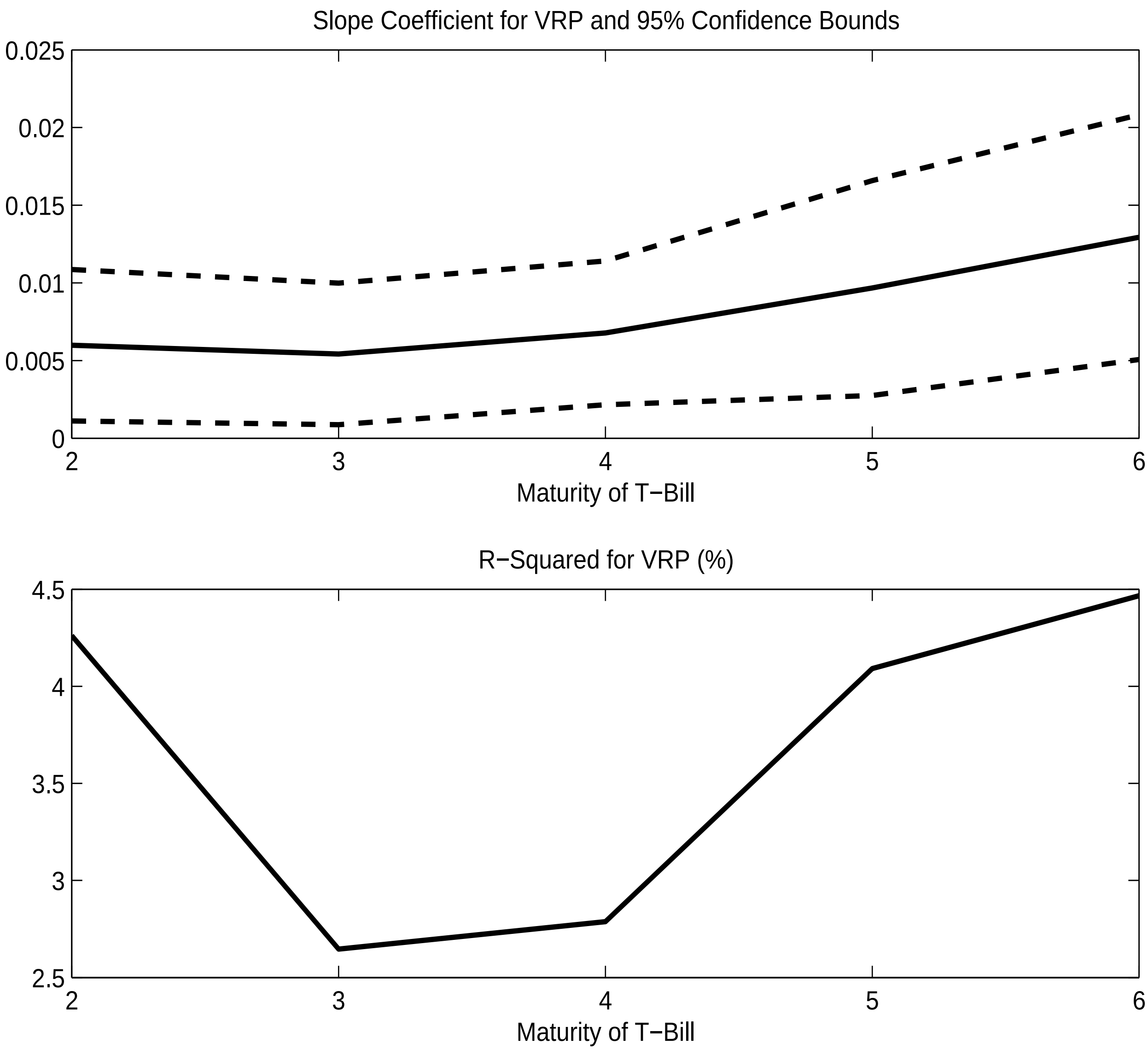

Figure 4 Estimated Slopes and R^2's of 2-6 Month T-bill Returns on VRP_t

The figure shows the estimated slope coefficients in the regressions of excess returns of 2-6 months t-bills over 1 month horizon on the variance risk premium variable. All of the regressions are based on monthly observations from January 1990 to December 2008.

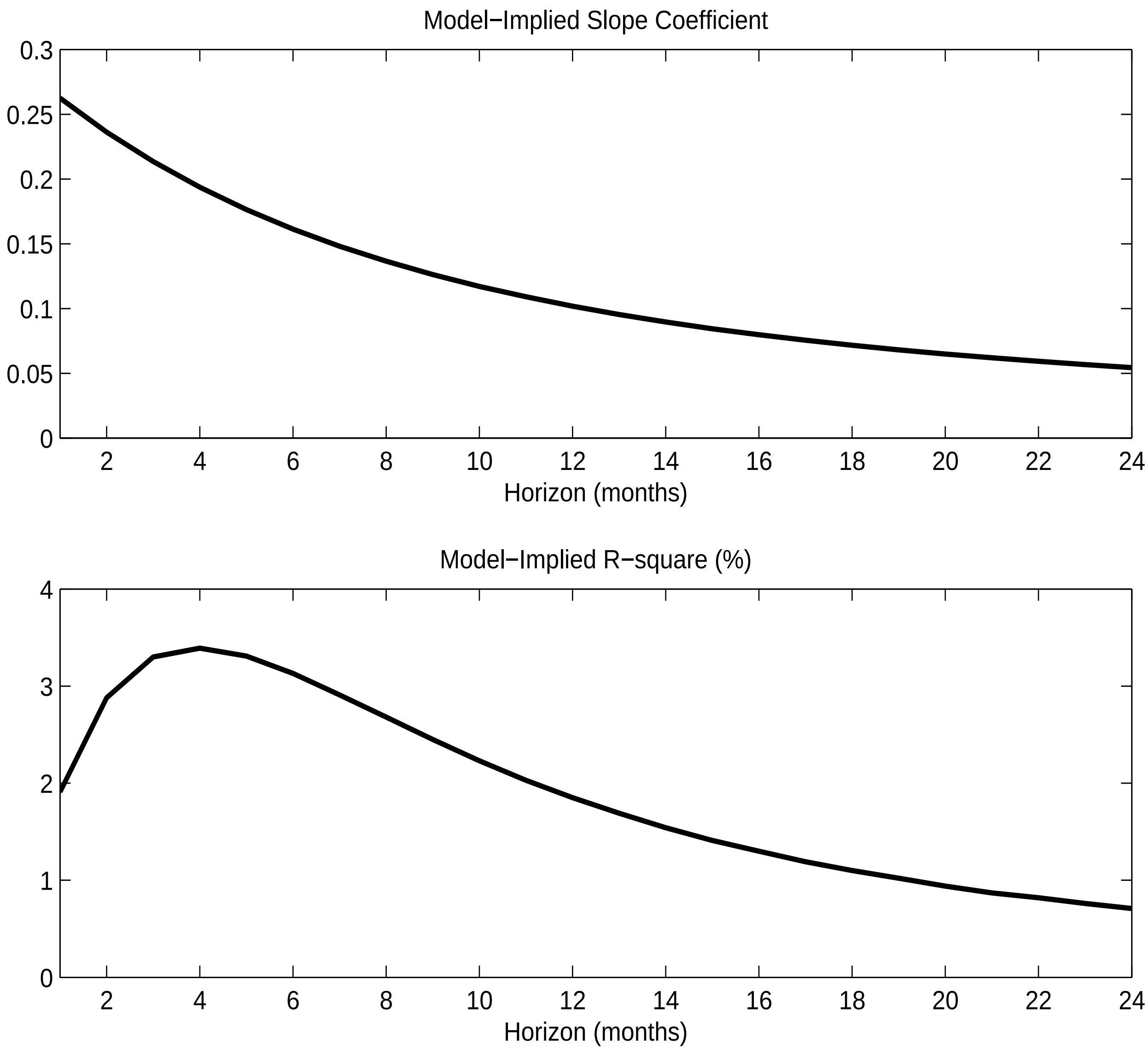

Figure 5 Model-Implied Slopes and R^2's of Equity Return Regression on VRP_t

The figure shows the population slope coefficients and R^2's from regressions of the scaled h-period equity excess returns on the variance risk premium variable implied by the equilibrium model.

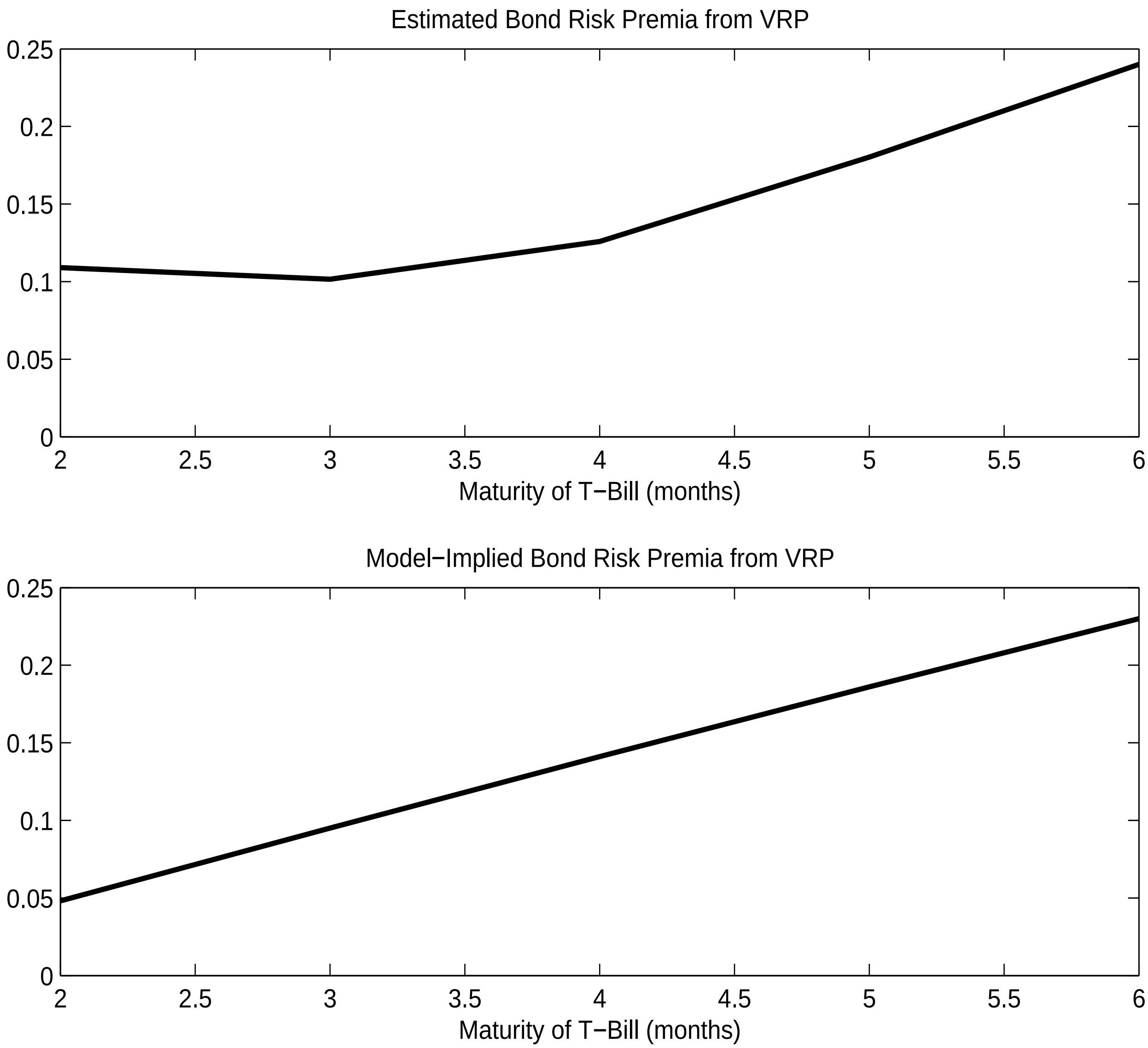

Figure 6 Estimated and Model-Implied Bond Risk Premia from VRP_t

The figure shows the estimated bond risk premia from regressing 2-6 month zero coupon bond excess returns on the variance risk premium variable (top panel) and the model-implied bond risk premia of 2-6 month zero coupon bond due to the economic uncertainty risk factor q_t (bottom panel).