Review of Finance, 2020, 45–98
doi: 10.1093/rof/rfz006
Advance Access Publication Date: 23 April 2019
OXFORD

Specification Analysis of Structural Credit Risk Models*

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Abstract

Empirical studies of structural credit risk models so far are often based on calibration, rolling estimation, or regressions. This paper proposes a GMM-based method that allows us to estimate model parameters and test model-implied restrictions in a unified framework. We conduct a specification analysis of five representative structural models based on the proposed GMM procedure, using information from both equity volatility and the term structure of single-name credit default swap (CDS) spreads. Our test results strongly reject the Merton (1974) model and two diffusion-based models with a flat default boundary. The other two models, one with jumps and one with stationary leverage ratios, do improve the overall fit of CDS spreads and equity volatility. However, all five models have difficulty capturing the dynamic behavior of both equity volatility and CDS spreads, especially for investment-grade names. On the other hand, these models have a much better ability to explain the sensitivity of CDS spreads to equity returns.

JEL classification: G12, G13, C51, C52

* The authors would like to thank Amit Goyal (the editor), an anonymous referee, Jean Helwege, Anh Le, George Tauchen, Marliese Uhrig-Homburg; seminar participants at Arizona, Central University of Finance and Economics, Cheung Kong Graduate School of Business, China Europe International Business School, Lehigh, MIT, Shanghai University of Finance and Economics; and conference participants (and especially discussants) at the 2006 Econometric Society North American Winter Meeting in Boston (Viral Acharya), the 2006 Derivatives and Risk Management Conference at FDIC, the 2007 Federal Reserve Board Conference on Credit Risk and Credit Derivatives, Moody's 4th Annual Credit Risk Conference in Copenhagen (Rangarajan Sundaram), the 2007 Gutmann Center Symposium on Credit Risk and the Management of Fixed-Income Portfolios in Vienna (Thomas Dangl), the 2007 China International Conference in Finance (Yi Zhou), the 2008 Mitsui Symposium at Michigan (Ren-Raw Chen), the 2008 Singapore International Conference on Finance (Jan Ericsson), the 2008 Northern Finance Association Meeting in Manitoba (Pascal Francois), and the 2009 American Finance Association Meeting in San Francisco (Ilya Strebulaev) for helpful comments and suggestions. We also thank Terry O'Brien for his editorial assistance. J.-Z.H. acknowledges a Smeal Summer Research Grant for partial support.

Keywords: Structural credit risk models, GMM, Consistent specification analysis, Credit default swaps, CDS hedging, Jump-diffusion models, Stochastic asset volatility, Realized equity volatility

Received April 1, 2017; accepted February 4, 2019 by Editor Amit Goyal.

1. Introduction

A widely used approach to credit risk modeling is the so-called structural method, originated from Black and Scholes (1973) and Merton (1974). A growing literature has empirically examined the implications of structural models for various financial variables, such as credit spreads (Eom, Helwege, and Huang, 2004), real default probabilities (Leland, 2004), both spreads and default rates (Huang and Huang, 2012), hedge ratios (Schaefer and Strebulaev, 2008), corporate bond return volatility (Bao and Pan, 2013), and prices of different (de facto) seniority levels (Bao and Hou, 2017). The main empirical methods used in this literature include calibration, rolling estimation, and regressions. Although these methods are intuitive, easy to implement, and widely used, it is known that, from a statistical point of view, they have some limitations.

In this study, we propose an alternative approach to testing structural credit risk models. More specifically, we construct a specification test based on certain model-implied variables, such as credit spreads and equity volatility. By assuming that both equity and credit markets are efficient and that the underlying structural model is correct, we obtain moment restrictions on model parameters (e.g., asset volatility and default boundary). We then use generalized method of moments (GMMs) of Hansen (1982) to conduct parameter estimation as well as a specification analysis of the structural model. Three aspects of this GMM-based specification test are worth noting. First, the test provides consistent econometric estimation of the model parameters. Second, the test allows us to conduct a precise inference on whether the model is rejected or not in the data. Third, the test is based on the joint behavior of time-series of asset dynamics and cross-sectional pricing errors for structural models.

For illustration, we apply the proposed approach to five affine, representative structural models of default that incorporate various economic considerations. For each of the five models, we construct its moment conditions using equity volatility and term structures of single-name credit default swap (CDS) spreads. We then test whether all the restrictions of the model are satisfied using the GMM, based on the model-implied CDS spreads and equity volatility. By minimizing the effect of measurement error from using firm characteristics, this test attributes the test results mostly to the specification error. Lastly, we examine the ability of the model to explain equity volatility, the CDS term structure, default rates, sensitivity of CDS spreads to equity returns, etc.

For the purpose of this study, using CDS data has at least two advantages over using corporate bond data. One is that CDS spread curves are readily available. The other is that in general the CDS market is more liquid than the corporate bond market. We include equity return volatility in moment conditions mainly because few empirical studies have examined the implications of structural models for this second moment variable.[1] In other

1 There is ample empirical evidence that individual equity volatility is time-varying and stochastic (see, e.g., the survey articles by Bollerslev, Chou, and Kroner, 1992; Bollerslev, Engle, and Nelson,

words, while equity volatility is usually used as an input in the empirical literature on structural models, this study treats equity volatility as an output of the models. Additionally, we use the so-called "model-free" realized equity volatility in our empirical analysis. As it is estimated using intraday high-frequency equity returns and involves no overlapping observations, realized volatility is more accurate than volatility estimates based on daily or monthly returns. Moreover, the use of the latter estimates implies that structural models are implicitly assumed to be able to fit perfectly the time series of equity volatility involving overlapping observations. Lastly, focusing on realized equity volatility is consistent with the evidence that volatility dynamics have a strong potential to help explain credit spreads (e.g., Zhang, Zhou, and Zhu, 2009).

For reasons of tractability and comparison, we focus on the Merton (1974) model and its four extensions with an exogenous default boundary in this study.[2] The four barrier-type models include the Black and Cox (1976) (BC) model with a flat default boundary, the Longstaff and Schwartz (1995) (LS) model with stochastic interest rates, the Collin-Dufresne and Goldstein (2001) (CDG) model with a stationary leverage, and the double-exponential jump diffusion (DEJD) model used in Huang and Huang (2002) and Kou (2002).[3]

We test each of the five models using a sample of 93 industrial companies in the USA that have a balanced panel of monthly realized equity volatility and CDS term structure over the period January 2002–December 2004. As the main purpose of our empirical analysis is to illustrate the proposed specification test of structural models, the choice of the sample period is not essential to the analysis. Nonetheless, this post dot-com bubble (and also post the Enron collapse) period includes many major corporate defaults and "actions." On the other hand, relatively "quiet" compared with the recent financial crisis, this sample period is less subject to illiquidity concern documented for the corporate bond market during the financial crisis (Dick-Nielsen, Feldhütter, and Lando, 2012; Friewald, Jankowitsch, and Subrahmanyam, 2012).

Our GMM-based specification tests strongly reject the Merton, BC, and LS models. The DEJD model is found to significantly outperform these three models. The CDG model is the best performing one among the five models: the model is not rejected by the GMM test for more than half of the 93 companies in our sample. Nonetheless, the fact that both the DEJD and CDG models are still rejected by a substantial number of firms in the sample indicates that something is missing in these models.

The pricing error results from the five models provide similar evidence. On the one hand, jumps and dynamic leverage help improve the model fit for investment-grade (IG) and high-yield (HY) names, respectively. On the other hand, the five models all substantially underestimate both equity volatility and CDS spreads for IG names during 2002 when credit risk is relatively high. In other words, these models have difficulty in capturing the

1994). This stylized fact should be taken into account in examining structural models that consider equity to be a contingent claim on the underlying firm asset value.

2 To be more precise, the Merton model implemented in this study is the "extended Merton model" tested in Eom, Helwege, and Huang (2004). A similar model is also studied in Bao and Pan (2013).

3 Kou (2002) develops the first DEJD-based equity option pricing model. Concurrently, Ramezani and Zeng (2007) use the DEJD to model individual stock returns. Huang and Huang (2002, 2012) provide the first application of the DEJD model in credit risk. Other examples using the DEJD-based structural model include Cremers, Driessen, and Maenhout (2008); Bao (2009); and Chen and Kou (2009).

dynamic behavior of both equity volatility and CDS spreads, especially for IG names—even though equity volatility in structural models is time-varying.

Interestingly, all five models, especially the Merton model, fare better in describing the sensitivity of CDS spreads to equity, in terms of the number of firms where the model-implied sensitivities are not rejected in our sample. Surprisingly, evidence from the actual hedging performance indicates that the Merton model outperforms the other four models.

To summarize, this study contributes to the credit risk literature by proposing and implementing a GMM-based specification test of structural models. Importantly, this approach, among other things, makes use of at least two advantages of GMM—its convenience and generality (see, e.g., Jagannathan, Skoulakis, and Wang, 2002). Our empirical findings (albeit based on a short sample) shed light on how to improve the existing structural models. Specifically, incorporating stochastic asset volatility and jumps into the Merton (1974) model may improve the ability of the model to predict not only CDS spreads and equity volatility but also hedge ratios of CDS spreads.[4]

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 outlines the class of structural models examined in our empirical analysis. Section 4 presents our econometric method of parameter estimation and specification tests. Section 5 describes the data used in our analysis, and Section 6 reports and discusses our empirical findings. Finally, Section 7 concludes.

2. Related Literature

Empirical studies of structural models go back to Jones, Mason, and Rosenfeld (1984), who implement a rolling estimation approach. Examples of following this approach include Eom, Helwege, and Huang (2004); Hull, Nelken, and White (2005); Arora, Bohn, and Zhu (2005); and Bao (2009). Huang and Huang (2002, 2012) propose a calibration approach with representative firms, which is also used in Chen, Collin-Dufresne, and Goldstein (2008); Schaefer and Strebulaev (2008); Du, Elkamhi, and Ericsson (2018); McQuade (2018); and Shi (2019). Regression-based studies include Collin-Dufresne, Goldstein, and Martin (2001) and Zhang, Zhou, and Zhu (2009). Ericsson and Reneby (2005) and Predescu (2005) combine a rolling estimation procedure with the maximum-likelihood estimation (MLE) method proposed in Duan (1994).

Among studies of structural models based on CDS data, Hull, Nelken, and White (2005) implement the Merton (1974) model using a calibration approach. Predescu (2005) examines the Merton model as well as a BC type barrier model. Chen *et al.* (2006) investigate the Merton, Black-Cox, and Longstaff-Schwartz models. Bao (2009) and Bai and Wu

4 Du, Elkamhi, and Ericsson (2018) incorporate stochastic volatility into the Merton (1976) jump-diffusion model and find that the resultant model of the unlevered asset return with stochastic volatility and jumps can jointly capture CDS spreads and option-implied volatilities; McQuade (2018) shows that combining stochastic volatility with endogenous default sheds light on many asset pricing anomalies, including the value premium, financial distress, and momentum puzzles. Both studies illustrate that a reasonable calibration for the variance risk premium allows their stochastic volatility models to match historical corporate yield spreads for medium and longer maturities, offering a potential resolution of the credit spread puzzle (à la Huang and Huang, 2002, 2012). Other recent studies of the puzzle include Bai, Goldstein, and Yang (2018); Feldhütter and Schaefer (2018); and Huang, Nozawa, and Shi (2018).

(2016) focus on the cross-section of spreads implied by structural models. Examples of studies that link CDS premiums with variables from structural models using a regression analysis include Ericsson, Jacobs, and Oviedo (2009) and Zhang, Zhou, and Zhu (2009).

This paper differs from the aforementioned studies in at least two aspects. First, it proposes and conducts a GMM-based specification test of structural models. In particular, equity volatility is treated as an output variable in the proposed test. Second, as a result, this study uses a different method for model parameter estimation. Consider, for example, asset volatility (a driving force behind the firm default risk). Estimates of this important parameter, used in the empirical analysis of structural models, include those calibrated to historical equity volatility and equity value (Jones, Mason, and Rosenfeld, 1984), option-implied equity volatilities (Hull, Nelken, and White, 2005), and default rates (Huang and Huang, 2012); those estimated using historical equity and bond return volatilities (Schaefer and Strebulaev, 2008); and those implied by corporate bond prices (Eom, Helwege, and Huang, 2004) or by CDS spreads (Kelly, Manzo, and Palhares, 2016). In our analysis, asset volatility is estimated using the GMM method with CDS term structures and realized equity volatility.

Our paper also fits in the literature on the implications of structural models for second moment variables (such as equity return volatility) as well as on their impact on credit risk. For instance, Campbell and Taksler (2003) find that idiosyncratic equity volatility can explain a significant part of corporate bond yield spreads cross-sectionally. Huang and Huang (2012) conjecture that a structural model with stochastic asset volatility and jumps may help solve the credit spread puzzle. Huang (2005) considers an affine class of structural models with both stochastic asset volatility and Lévy jumps. Based on regression analysis, Zhang, Zhou, and Zhu (2009) provide empirical evidence that a stochastic asset volatility model may improve the model performance. Perrakis and Zhong (2015) extend the Leland and Toft (1996) model to allow for constant elasticity of variance. Kelly, Manzo, and Palhares (2016) provide more recent evidence of stochastic asset volatility; see also Du, Elkamhi, and Ericsson (2018) and McQuade (2018). In a closely related study, Bao and Pan (2013) focus on corporate bond return volatility and document that the volatility implied from the Merton (1974) model with stochastic interest rates underestimates substantially the observed corporate bond return volatility.

The literature on hedge ratios implied by structural models goes back to Schaefer and Strebulaev (2008), who find that on average, the Merton model-implied sensitivity of a firm's corporate bond returns to its equity returns is not statistically different from the in-sample empirically estimated hedge ratios. Bao and Hou (2017) investigate how a corporate bond's position in its issuer's maturity structure affects its sensitivity to the issuer's equity return. They show that both the direction and the magnitude of this *de facto* seniority effect are consistent with what are implied from an extended Merton model. Huang and Shi (2016) document that on average, the Merton model also captures the in-sample sensitivity of corporate bond spreads to equity returns. In addition, they examine the actual hedging performance of model-implied sensitivities of both corporate bond returns and spreads, thereby providing an out-of-sample test of the explanatory power of hedging portfolios. On the other hand, focusing on pairs of stock returns and CDS spread changes with the same underlying over a short interval (e.g., 5 days), Kapadia and Pu (2012) find that about 41% of stock returns are associated with CDS spread changes in the same direction, as opposed to the prediction of the Merton model. This discrepancy is shown to reflect an imperfect equity-credit market integration at short horizons. Huang, Rossi, and Wang

(2015) find similar results based on pairs of stock and corporate bond returns and also provide evidence that equity market sentiment helps improve the equity-credit market integration, especially after the financial crisis.

In this study we examine not only hedge ratios of CDS spreads but also actual hedging performance of structural models. In addition, we go beyond the Merton model.

As mentioned before, we use CDS data instead of corporate bond data in our empirical analysis, partly to avoid the liquidity problem in the latter market. For evidence on corporate bond illiquidity, see Bao, Pan, and Wang (2011); Bongaerts, de Jong, and Driessen (2017); Chen, Lesmond, and Wei (2007); Das and Hanouna (2009); Han and Zhou (2016); Helwege, Huang, and Wang (2014); Longstaff, Mithal, and Neis (2005); Mahanti *et al.* (2008); Schestag, Schuster, and Uhrig-Homburg (2016), among others. In addition, using CDS term structures facilitates the implementation of the proposed GMM-based test—it is known that data on term structures of corporate bond spreads are not easily available for individual firms. For a recent survey on the CDS market, see Augustin *et al.* (2016).

Lastly, note that there is a large theoretical literature on structural credit risk modeling (see, e.g., Huang and Huang, 2012; Sundaresan, 2013, and references therein), although for tractability and comparison we consider only five structural models in our empirical analysis. For example, the class of endogenous-default models, not considered in this paper, includes those without strategic default, such as Geske (1977) and Leland and Toft (1996), and strategic default models, such as Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Acharya and Carpenter (2002), and Acharya *et al.* (2006, 2019). Strategic default models of perpetual bonds are considered in Huang and Huang (2012). Endogenous default models with finite maturity of Geske (1977) and Leland and Toft (1996) are examined in Eom, Helwege, and Huang (2004). Another example not covered in this paper is the Duffie and Lando (2001) model with incomplete accounting information. Additionally, François and Morellec (2004) examine the impact of the US bankruptcy procedure on risky debt prices. He and Xiong (2012) and He and Milbradt (2014) consider both rollover risk and corporate bond illiquidity.

3. Affine Structural Credit Risk Models

In this section, we first review the five structural models to be tested in our specification analysis. We then discuss the model implications for CDS spreads, equity volatility, and sensitivities of CDS spreads to equity return.

3.1 Models

Although the five models differ in certain economic assumptions, they all belong to the class of affine structural credit risk models and can be considered to be different specifications of one single model.

Let V be the firm's asset process, K the default boundary, and r the default-free interest rate process. Assume that, under a risk-neutral measure $\mathbb{Q}$,

$$\frac{\mathrm{d}V_t}{V_{t-}} = (r_t - \delta)\mathrm{d}t + \sigma_v \mathrm{d}W_t^{\mathbb{Q}} + \mathrm{d}\left[\sum_{i=1}^{N_t^{\mathbb{Q}}}\left(Z_i^{\mathbb{Q}} - 1\right)\right] - \lambda^{\mathbb{Q}}\xi^{\mathbb{Q}}\mathrm{d}t, \tag{1}$$

$$\mathrm{d}\ln K_t = \kappa_\ell\left[-\nu - \phi(r_t - \theta_r) - \ln\left(K_t/V_t\right)\right]\mathrm{d}t, \tag{2}$$

$$\mathrm{d}r_t = (\alpha - \beta r_t)\mathrm{d}t + \sigma_r \mathrm{d}W^{Q}_{rt}, \tag{3}$$

where δ, σ_v, κ_ℓ, ν, ϕ, α, β, σ_r, and $\theta_r = \alpha/\beta$ are constants, and W^Q and W^Q_r are both one-dimensional standard Brownian motion under the risk-neutral measure and are assumed to have a constant correlation coefficient of ρ. In Equation (1), the process N^Q is a Poisson process with a constant intensity $\lambda^Q > 0$, the Z^Q_is are i.i.d. random variables, and $Y^Q \equiv \ln(Z^Q_1)$ has a double-exponential distribution with a density given by

$$f_{Y^Q}(y) = p^Q_u \eta^Q_u \mathrm{e}^{-\eta^Q_u y} 1_{\{y \geq 0\}} + p^Q_d \eta^Q_d \mathrm{e}^{\eta^Q_d y} 1_{\{y < 0\}}. \tag{4}$$

In Equation (4), parameters $\eta^Q_u, \eta^Q_d > 0$ and $p^Q_u, p^Q_d \geq 0$ are all constants, with $p^Q_u + p^Q_d = 1$. The mean percentage jump size ξ^Q is given by

$$\xi^Q = \mathrm{E}^Q\left[\mathrm{e}^{Y^Q} - 1\right] = \frac{p^Q_u \eta^Q_u}{\eta^Q_u - 1} + \frac{p^Q_d \eta^Q_d}{\eta^Q_d + 1} - 1. \tag{5}$$

All five models are special cases of the general specification in Equations (1)–(5). For instance, if the jump intensity is zero, then the asset process is a geometric Brownian motion. This specification is used in the four diffusion models, namely, the models of Merton (1974), BC, LS, and CDG.

Regarding the specification of the default boundary K, it is a point at the bond maturity in the (original) Merton model and a discrete barrier in the extended Merton model. If κ_ℓ is set to be zero, then the default boundary is flat (a continuous barrier), an assumption made in the BC, LS, and the DEJD models.

If α, β and σ_r in Equation (3) are zero, then the interest rate is constant. This leads to the three one-factor models: the Merton, BC, and DEJD models. If both β and σ_r are greater than zero, then we have the two-factor models, LS and CDG, where the dynamics of the risk-free rate follow the Vasicek model specified in Equation (3). Additionally, the CDG model assumes that $\kappa_\ell > 0$ and that the default boundary follows the mean-reverting specification in Equation (2).

Lastly, we obtain the DEJD model if the jump intensity is strictly positive, the risk-free rate is constant, and the default boundary is flat.

We assume a constant recovery rate for comparison with other studies and also because the CDS database that we use includes recovery rate estimates for each CDS contract.

3.2 Valuation of Single-Name CDS Contracts

Under each of the five structural models, it is straightforward to calculate the CDS spread. Let $Q(0, T)$ denote the survival probability over $(0, T]$ under the T-forward measure. Then the CDS spread of a T-year CDS contract is given by

$$\mathrm{cds}(0, T) = \frac{(1 - R)E^Q\left[\mathrm{e}^{-\int_0^\tau r(u)\mathrm{d}u} I_{\{\tau < T\}}\right]}{\sum_{i=1}^{4T} B(0, T_i)Q(0, T_i)/4}, \tag{6}$$

where R is the recovery rate, $B(0, \cdot)$ the default-free discount function, $\tau > 0$ the default time, $I_{\{\cdot\}}$ the indicator function, and $E^Q[\cdot]$ the expectation under the risk-neutral measure. To simplify the computation, we follow the literature to make the standard assumption that the settlement of the contract occurs on the next payment day. It then follows from Equation (6) that

$$\mathrm{cds}(0,T) = \frac{(1-R)\sum_{i=1}^{4T} B(0,T_i)[Q(0,T_{i-1}) - Q(0,T_i)]}{\sum_{i=1}^{4T} B(0,T_i)Q(0,T_i)/4}. \quad (7)$$

As a result, the implementation of a structural model amounts to the calculation of the survival probability $Q(0,\cdot)$. In the Merton (1974) and BC models, $Q(0,\cdot)$ has closed-form solutions. The survival probabilities in the LS, CDG, and DEJD models do not have known closed-form solutions but can be calculated numerically (see, e.g., Huang and Huang, 2012, for details).

In addition to CDS spreads, other model-implied credit market variables include CDS spread changes, CDS volatilities, corporate bond return volatilities, etc. However, corporate bond volatilities have a sizable illiquidity component and CDS volatilities might also be a bit high compared with fundamentals (Bao and Pan, 2013; Bao *et al.*, 2015). Therefore, given the purpose of this study, we do not consider these second moment variables in credit markets in our empirical analysis.

3.3 Equity Market Variables

In this subsection we focus on more liquid equity market variables, which have received relatively little attention in the empirical literature on structural models.

Consider equity return volatility first. As pointed out by Merton (1974), the function relating the equity volatility and asset volatility is also model-dependent

$$\sigma_E(t) = \sigma_v \frac{V_t}{E_t}\frac{\partial E_t}{\partial V_t}, \quad (8)$$

where E_t is the time-t equity value, and Equation (8) applies to equity volatility of the continuous diffusion component for the DEJD model. Note that $\sigma_E(t)$ is time-varying even if σ_v is assumed to be constant.

Next, we consider comovements between CDS and equity, in order to better understand their relative pricing as well as how to hedge their common exposures across markets. Following Schaefer and Strebulaev (2008), we express the sensitivity of a CDS spread to the underlying equity return in terms of their partial derivatives with respect to the underlying firm value

$$\Delta_{E,t}^{\mathrm{cds}} \equiv \frac{\partial \mathrm{cds}(t,T)}{\partial E_t/E_t} = \frac{\partial \mathrm{cds}(t,T)/\partial V_t}{\partial E_t/\partial V_t} E_t. \quad (9)$$

As illustrated in Sections 4.3 and 6.5, both $\partial\mathrm{cds}(t,T)/\partial V_t$ and $\partial E_t/\partial V_t$ are functions of $\partial Q(t,\cdot)/\partial V_t$. As such, once $Q(t,\cdot)$ is known, $\Delta_{E,t}^{\mathrm{cds}}$ can be calculated using Equation (9).

Unlike its counterpart for corporate bonds, the hedge ratio for a CDS contract is not the same as its sensitivity to equity. Instead, the latter hedge ratio is defined as the dollar change in the value of the CDS contract for each percentage change in the equity value

$$h_{E,t}^{\mathrm{cds}} \equiv \frac{\partial V_t^{\mathrm{cds}}}{\partial E_t/E_t} = \frac{\partial \mathrm{cds}(t,T)}{\partial E_t} E_t D_t^{\mathrm{cds}}, \quad (10)$$

where V_t^{cds} denotes the time-t value of a CDS contract with a notional of \$10 million, and $D_t^{\text{cds}} = \sum_{i=1}^{4T} B(t, T_i)Q(t, T_i) \times 2.5$ million is defined as the change in the mark to market value (in million) for each unit of change in the quoted spread.[5]

4. A Specification Test of Structural Models

In this section, we propose a specification test of structural models under the GMM framework of Hansen (1982). We first review the framework albeit using moment conditions pertinent to structure models. We then discuss finite-sample properties of GMM. Lastly, we focus on the implementation of the proposed specification test.

4.1 GMM Estimation of Structural Credit Risk Models

As mentioned before, the fundamental pricing relationship implied by a structural model has implications for credit spreads, equity volatility, default probabilities, leverage, corporate bond returns, corporate bond return volatility, hedge ratios, etc. To evaluate the model, we first estimate the model parameters that may include asset volatility, default boundary, asset jump intensity, or dynamic leverage coefficients. Let θ denote the vector of the model parameters to be estimated and $\hat{\theta}$ the estimated vector. We then take $\hat{\theta}$ as given and examine the pricing performance of the (estimated) model. Below we describe how to implement this idea using GMM, following largely Cochrane (2009).

As noted before, we focus on model-implied CDS spreads and equity volatility in the empirical analysis. Let $\text{cds}(t, t+T_m)$ and $\sigma_E(t)$ be the time-t CDS spread with maturity $t+T_m$ and equity volatility under a given structural model, specified in Equations (7) and (8), respectively. Let $\widetilde{\text{cds}}(t, t+T_m)$ and $\widetilde{\sigma_E}(t)$ be the time-t observed counterparts of $\text{cds}(t, t+T_m)$ and $\sigma_E(t)$. Consider the following vector of pricing errors (so-called moment conditions):

$$f(\theta, t) = \begin{bmatrix} \widetilde{\text{cds}}(t, t+T_1) - \text{cds}(t, t+T_1) \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \widetilde{\text{cds}}(t, t+T_M) - \text{cds}(t, t+T_M) \\ \widetilde{\sigma_E}(t) - \sigma_E(t) \end{bmatrix}, \tag{11}$$

where M denotes the number of CDS contracts with different maturities and the same underlying firm. Under the null hypothesis that the model is correctly specified, we have

$$E[f(\theta, t)] = 0. \tag{12}$$

To test the above hypothesis, we construct a time series of $f(\theta, t)$ over the sample period and consider its time-series mean in the following:

$$g(\theta, T) \equiv \frac{1}{T} \sum_t^T f(\theta, t). \tag{13}$$

In other words, $g(\theta, T)$ represents the sample mean of the moment conditions. If $M = \dim(\theta) - 1$ (i.e., the number of moment conditions is the same as the number of parameters to be estimated), then we can pick θ such that $g(\theta, T) = 0$. In general, however,

5 We use the ISDA CDS Standard model to mark a given CDS contract to market. Documentation of the model as well as the source code for the model is available at www.cdsmodel.com.

$M > \dim(\theta) - 1$ as in our case; that is, there are more moment conditions than parameters. In this case, we can pick θ such that linear combinations of the moment conditions are zero. This is a challenging task, however, especially given that both CDS spreads and equity volatility are allowed to be observed with measurement errors in this analysis. As such, we choose θ to minimize a quadratic function of the pricing errors. Doing so leads to the so-called GMM estimator:

$$\hat{\theta} = \arg\min g(\theta, T)' W(T) g(\theta, T), \tag{14}$$

where $W(T)$, a weighting matrix, denotes the asymptotic covariance matrix of $g(\theta, T)$ (Hansen, 1982). With mild regularity conditions, $\hat{\theta}$ is $\sqrt{T}$-consistent and asymptotically normally distributed, under the null hypothesis.

Furthermore, we implement the iterative GMM. That is, we begin with $W(T) = I$, the identity matrix, and estimate θ. Next, we use a heteroscedasticity robust estimator for the variance–covariance matrix $W(T)$ that allows for autocorrelation in the errors (Newey and West, 1987), and obtain a new $\hat{\theta}$. We repeat this procedure until it converges.

Given $\hat{\theta}$ that minimizes the quadratic form specified in Equation (14), we can then examine how well the candidate model fits. If the pricing errors are "large" under the appropriately defined GMM metric, the candidate model specification will be rejected. Formally, we conduct the following test:

$$J_T = T \min_{\theta} g(\theta, T)' W(T) g(\theta, T) \sim \chi^2(N^{\text{oi}}), \tag{15}$$

where $N^{\text{oi}} = M + 1 - \dim(\theta)$, the degree of freedom of the χ^2-distribution, equals the number of overidentifying moment conditions. As a result, the GMM J_T-test allows for an omnibus test of the overidentifying restrictions.

4.2 Finite-Sample Properties of GMM

The J_T-test specified in Equation (15) is an asymptotic test. Several studies have examined finite-sample properties of GMM estimators applied to asset pricing models, although the literature has focused mainly on consumption-based models and linear factor models in the equity market (see, e.g., Hall, 2005, and references therein). For instance, Tauchen (1986) considers the Hansen and Singleton (1982) consumption-based asset pricing model and examines the behavior of the two-step GMM estimator using one asset in the estimation. He finds that the bias of the estimator tends to increase as the degree of overidentification (N^{oi}) increases but the empirical sizes of the J_T-test tend to be close to the asymptotic value. Kocherlakota (1990) extends the analysis of Tauchen (1986) to multiple assets and his findings suggest that the iterated GMM estimator considerably improves the finite-sample behavior of GMM. Using predictive regression models for stock returns, Ferson and Foerster (1994) find that while sizes of the two-step GMM-based J_T statistics are often too large with finite samples, the iterated GMM approach has superior finite-sample properties. Hansen, Heaton, and Yaron (1996) consider a consumption-based asset pricing model where the representative agent's utility function allows for time non-separability. They find that when the number of the overidentifying restrictions is high (five), the asymptotic theory is far from the finite-sample property. Lettau and Ludvigson (2001) argue that the one-stage GMM is more appropriate than the two-stage GMM with an estimated weighting matrix in the application pursued in their study—where the time-series sample is small relative to the cross-sectional sample size.

In our specification analysis, we test a given candidate model firm by firm. Based on the insights from the aforementioned studies, in order to mitigate the potential small sample problems in our tests, we need to keep the degree of overidentification minimal. As discussed in Section 4.3, for a given firm, the number of parameters to be estimated using the GMM ranges from one for the Merton model to four for the CDG model. As such, we use four CDS contracts and realized equity volatility (i.e., five moment conditions) with 36 monthly observations in each GMM test. That is, the degree of overidentification ranges from one in CDG to four in Merton in our tests. As a robustness check, we also test the Merton model using only one CDS contract and realized equity volatility such that the degree of overidentification is one. The number of time-series observations relative to the number of moment conditions is reasonably large, given that the latter is no more than five in our tests. Additionally, we implement the iterative GMM. Taken together, the findings of the aforementioned studies based on the equity market suggest that small sample problems are not a major concern in our GMM tests.

4.3 Implementation

In this subsection, we discuss the implementation of the proposed GMM specification test. First, to make the estimation tractable, we estimate the interest rate process separately from firm-specific model parameters for the two models with stochastic interest rates. This is a reasonable strategy, since the interest rate parameters are common inputs in these models and those firm-specific parameters do not affect the interest rate process.

We use the 3-month LIBOR as a proxy for the short rate (r_t) in the estimation. We estimate the interest rate volatility using $\hat{\sigma}_r = \text{stdev}(r_t)$. Given that the one-factor Vasicek (1977) model is a crude approximation to the observed term structure dynamics, we opt to estimate the risk-neutral drift parameters, α and β, month-by-month as follows:

$$\left\{\hat{\alpha}_t, \hat{\beta}_t\right\} = \arg\min \sum_{T=T_1}^{T_6} \left[y^{\text{data}}_{t,t+T} - y_{t,t+T}(\alpha, \beta)\right]^2,$$

where the term structure of observed interest swap rates used in the above nonlinear least square estimation is $y^{\text{data}}_{t,t+T}$ with $T = 1, 2, 3, 5, 7,$ and 10 years—matching CDS maturities included in our sample (see Section 5.1). The cross-sectional pricing errors of the Vasicek model range from 12 to 112 basis points (bps) during the full sample period. The sample means of monthly estimates ($\hat{\beta}_t, \hat{\sigma}_{r,t}$), which are obtained by rolling-window estimations, are 0.3820 and 0.0156, respectively; while $\hat{\beta}_t$ is larger than those reported in previous studies based on much longer samples (Schaefer and Strebulaev, 2008; Bao and Pan, 2013), the magnitude of $\hat{\sigma}_{r,t}$ is consistent with their estimates.

Next, we focus on those firm-specific model parameters. For ease of reference, let θ denote the vector of these parameters in the discussion that follows—namely, θ does not include $(\alpha, \beta, \sigma_r)$. For a given structural model, we estimate its θ in two steps.

In Step 1, fixing an initial θ, we calculate the month-t model-implied CDS spreads, $\text{cds}(t, \cdot) = \left\{\text{cds}(t, t+T_j)\right\}_{j=1}^{M}$, and the model-implied equity volatility, $\sigma_E(t)$, using Equations (7) and (8), respectively.[6] Given the model-implied $\text{cds}(t, \cdot)$ and $\sigma_E(t)$, we then

6 In connection with Equation (8), we implicitly include the empirically observed (quasi-market) leverage ratio as one moment condition by imposing the following constraint during the estimation: at the end of each month, for every firm in our sample, we adjust the coupon rate of its debt such that

compute the month-t vector $f(\theta, t)$ of pricing errors defined in Equation (11). Repeating this for every month, we obtain a time series of vector $f(\theta, t)$ as well as its sample mean, $g(\theta, T)$ in Equation (13), over the full sample period.

In Step 2, we solve the optimization problem specified in Equation (14), where the weighting matrix $W(T)$ is estimated iteratively—and in each iteration we use the Newey–West auto-correlation robust estimator of the covariance matrix with three lags.

In the two-step procedure outlined above, one key component is the choice of the initial θ. In the case of the Merton model, $\theta = (\sigma_v)$. The initial $\sigma_v = \sigma_E L_q$, where the quasi market leverage ratio $L_q = F/(F + E)$, F denotes the total debt (book value) and E the market equity value. In the case of the Black–Cox model, $\theta = (\sigma_v, K)$. The initial σ_v is the GMM estimate of σ_v obtained using the Merton model. We set the initial K/F to 1.2 if $\bar{L} < 0.2$; 1 if $0.2 < \bar{L} \leq 0.4$; 0.8 if $0.4 < \bar{L} \leq 0.6$; 0.6 if $0.6 < \bar{L} \leq 0.8$; and 0.4 if $\bar{L} > 0.8$, where $\bar{L}$ is the firm's mean leverage ratio over the full sample period. Such choice of the initial (σ_v, K) is also followed in the estimation of the LS, CDG, and DEJD models.

In the case of the two-factor LS model, we need to estimate (σ_v, K, ρ), where the initial correlation coefficient ρ used is the correlation between equity returns and the interest rate. Estimates of ρ obtained in the literature, however, are usually zero or slightly negative (see, e.g., Eom, Helwege, and Huang, 2004; Schaefer and Strebulaev, 2008; Bao and Pan, 2013). Therefore, we restrict ρ to be zero in the estimation of the LS model. As a result, $\theta = (\sigma_v, K)$ in this case.

The other two-factor model, the CDG model, involves five parameters: $(\sigma_v, \rho, \kappa_\ell, \nu, \phi)$. Results from an untabulated analysis indicate that coefficients ρ and ϕ seem difficult to be simultaneously identifiable and that ρ is not bounded between -1 and $+1$. As a result, we impose the restriction that $\rho = 0$. Doing so also makes it easier to see the incremental impact of the stationary leverage ratio relative to the LS model. It follows that the vector of parameters to be estimated using GMM is $\theta = (\sigma_v, \kappa_\ell, \nu, \phi)$. The initial values of κ_ℓ, ν, ϕ are chosen to be the same as the values calibrated in CDG.

In the case of the DEJD model, the model parameters include $\left(\sigma_v, K, \lambda^Q, p_u^Q, \eta_u^Q, \eta_d^Q\right)$. The latter three parameters, $(p_u^Q, \eta_u^Q, \eta_d^Q)$, however, enter the solution function multiplicatively with λ^Q as in $\lambda^Q \xi^Q$ and, as a result, are very difficult to identify in our GMM estimator. To overcome this technical difficulty in the GMM estimation of the DEJD model, we let $\theta = \left(\sigma_v, K, \lambda^Q\right)$, and restrict the domain of $(p_u^Q, \eta_u^Q, \eta_d^Q)$ to the following particular values: $p_u^Q \in \{0.25, 0.5, 0.75\}$, $\eta_u^Q \in \{3, 5\}$, and $\eta_d^Q \in \{3, 5\}$, where the inputs of (η_u^Q, η_d^Q) are motivated by the calibration exercise of Huang and Huang (2002). In our estimation, for each firm, we choose the particular set of jump parameters with the smallest J-statistic as the "best" jump model estimate. The initial λ^Q is set to 0.1.

5. Data Description

Data used in this study include single-name CDS spreads, data on intraday equity returns (used to estimate realize equity volatility), firm balance sheet information, and risk-free interest rates. In this section, we describe each of these four data sets in detail, and then present summary statistics on CDS spreads and firm characteristics.

it is valued at par and, as a result, that the market value of the firm is equal to (market equity + book debt).

5.1 CDS Spreads

We use CDS data from Markit, a comprehensive data source that assembles a network of industry-leading partners who contribute information across several thousand credits on a daily basis. Based on the contributed quotes, Markit creates daily composite quotes for each CDS contract, which must pass the stale data test, flat curve test, and outlying data test. Together with the pricing information, the Markit data set also reports average recovery rates used by data contributors in pricing each CDS contract. In addition, an average of Moody's and S&P ratings is also included.

We begin with collecting all CDS quotes written on US entities (sovereign entities excluded) and denominated in US dollars. Following previous empirical studies on structural models (e.g., Eom, Helwege, and Huang, 2004), we exclude financial and utility sectors from the sample. In addition, we focus on senior unsecured CDS contracts and exclude the subordinated class of CDS contracts. Furthermore, we limit our sample to CDS contracts with modified restructuring clauses, as they are the most traded in the US market.

For the purpose of GMM estimation, we restrict the sample to those CDS names with at least 36 consecutive monthly observed spreads available. We then match the remaining data to the Center for Research in Security Prices (CRSP), NYSE Trade and Quote (TAQ), and Compustat data bases. Note that the NYSE TAQ data base includes intraday (tick-by-tick) transaction data for all securities listed on NYSE, AMEX, and NASDAQ. The final sample of entities includes 93 firms, which are listed in Appendix Table AI.

The Markit data set has single name CDS spreads available for maturities of 0.5, 1, 2, 3, 5, 7, 10, 15, 20, and 30 years. Due to the liquidity concern and missing values, we focus on CDS spreads with maturities of 1–10 years. For each entity, we create the monthly CDS spread by selecting the latest composite quote in each month, and, similarly, the monthly recovery rates linked to CDS spreads. Our final sample includes 93 single names with monthly CDS spreads for maturities of 1, 2, 3, 5, 7, and 10 years over the period January 2002–December 2004.

5.2 Equity Volatility from High Frequency Data

By the theory of quadratic variation, it is possible to construct increasingly accurate measure for the model-free realized volatility or average volatility, during a fixed time interval (say, a day or a month), by summing increasingly finer sampled squared high-frequency returns (Andersen *et al.*, 2001; Barndorff-Nielsen and Shephard, 2002; Meddahi, 2002). In testing structural models, asset return volatility is often backed out from equity return volatility (e.g., Jones, Mason, and Rosenfeld, 1984; Eom, Helwege, and Huang, 2004); as such, estimation of asset volatility benefits from more accurate estimates of equity volatility.

By definition, intraday returns of a given stock on day t are given by

$$r^s_{t,i} \equiv \ln S_{t,i\cdot\Delta} - \ln S_{t,(i-1)\cdot\Delta}, \tag{16}$$

where S_{t,t_i} denotes the time-t_i stock price on day t, $r^s_{t,i}$ refers to the ith within-day return on day t, and Δ is the sampling frequency and chosen to be 5-min. The realized equity volatility (squared) for period t is

$$\widetilde{\sigma_E}(t)^2 \equiv \sum_{i=1}^{1/\Delta} \left(r^s_{t,i}\right)^2, \tag{17}$$

which converges to the integrated or average variance during period t. For a jump-diffusion model, the continuous component of equity volatility (squared) can be estimated with the so-called "bi-power variation" as follows:

$$\widetilde{\sigma_E}(t)^2 \equiv \frac{\pi}{2}\frac{1/\Delta}{1/\Delta - 1}\sum_{i=2}^{1/\Delta}|r^s_{t,i-1}||r^s_{t,i}|\,. \tag{18}$$

As shown by Barndorff-Nielsen and Shephard (2004), such an estimator of realized equity volatility is robust to the presence of rare and large jumps.

Realized equity volatilities used in our analysis are estimated using TAQ data. The monthly realized variance is the sum of daily realized variances, constructed from the squares of intraday 5-min returns. Then, monthly realized volatility is just the square-root of the annualized monthly realized variance.

5.3 Capital Structure and Asset Payout

Assets and liabilities are key variables in evaluating structural models of credit risk. The accounting information is obtained from Compustat on a quarterly basis and assigned to each month within the quarter. We calculate the firm asset as the sum of total liabilities plus market equity, where the market equity is obtained from the monthly CRSP data on shares outstanding and equity prices. Leverage ratio is estimated by the ratio of total liabilities to the firm's assets. The asset payout ratio is estimated by the weighted average of the interest expense and dividend payout. Both ratios are reported as annualized percentages.

5.4 Risk-Free Interest Rates

Default-free interest rates used in the calculation of CDS spreads are estimated from the 3-month LIBOR and interest rate swaps with maturities of 1, 2, 3, 5, 7, and 10 years. These data are available from the Federal Reserve H.15 Release.

5.5 Summary Statistics

Table I provides summary statistics on firm characteristics and CDS spreads across either rating categories (panel A) or sectors (panel B). As can be seen from panel A1, our sample is concentrated in A-rated (25) and BBB firms (45), which account for 75% of the full sample, reflecting the fact that contracts on IG names dominate the CDS market. In terms of the average over both the time-series and cross-section in our sample, the 5-year CDS spread is 144 bps with a standard deviation of 3.18%, equity volatility 38.40% (annualized), the leverage ratio 48.34%, asset payout ratio 2.14%, and the quoted recovery rate 40.30%. As expected, the CDS spread, equity volatility, and leverage ratio all increase as the credit rating deteriorates. The recovery rate largely decreases as the rating deteriorates but has low variations.

Figure 1 plots both the term structure (from 1 to 10 years) and time evolution of the average CDS spreads over the full sample period January 2002–December 2004. Clearly, the average spreads show large variations and have a peak around late 2002.

Figure 2 plots both the 5-year CDS spreads (top panel) and equity volatility (bottom panel) by three different rating groups (AAA–A, BBB, and BB–CCC) over the full sample period. An inspection of the figure indicates that CDS spreads and equity volatilities appear to move together sometime during market turmoils but are only loosely related during quiet periods. The 5-year CDS spreads clearly have a peak in late 2002 across all three rating

Table I. Summary statistics on single-name corporate CDS spreads

This table reports summary statistics on the full sample of 93 single-name corporate CDS contracts, by either credit ratings (Panel A) or sectors (Panel B). Rating is the average of Moody's and Standard and Poor's ratings. Equity volatility is estimated using 5-min intraday returns. Leverage ratio is total liabilities divided by the total asset (total liability plus market equity). Asset payout ratio is the weighted average of dividend payout and interest expense over the total asset. Recovery rate is the quoted recovery rate accompanied with the CDS premium from the dealer-market. The full sample includes CDS spreads with 1-, 2-, 3-, 5-, 7-, and 10-year maturities over the period from January 2002 to December 2004.

Panel A1: Firm characteristics by credit ratings

Credit rating	Sample firms		Equity volatility (%)	Leverage ratio (%)	Asset payout (%)	Recovery rate (%)
	Number	Percentage				
AAA	1	1.08	36.36	63.71	2.22	40.88
AA	6	6.45	31.50	20.92	1.53	40.92
A	25	26.88	32.51	38.15	2.02	40.57
BBB	45	48.39	35.54	51.84	2.26	40.73
BB	11	11.83	47.19	57.76	2.15	39.51
B	4	4.30	83.23	72.61	2.28	38.23
CCC	1	1.08	81.94	93.93	2.89	26.57
Overall	93	100.00	38.40	48.34	2.14	40.30

Panel A2: Average CDS spreads (%) by CDS maturities and ratings

	Maturity of CDS					
	1-Year	2-Year	3-Year	5-Year	7-Year	10-Year
AAA	0.23	0.28	0.32	0.43	0.45	0.49
AA	0.12	0.13	0.15	0.20	0.23	0.28
A	0.25	0.29	0.32	0.39	0.43	0.49
BBB	0.74	0.79	0.86	0.94	0.98	1.05
BB	2.62	2.74	2.84	2.90	2.92	2.92
B	7.52	7.20	7.51	7.25	7.01	6.79
CCC	25.26	22.99	20.91	18.81	18.03	17.31
Overall	1.34	1.36	1.40	1.44	1.45	1.49

Panel A3: Standard deviation of CDS spreads (%) by CDS maturities and ratings

AAA	0.17	0.19	0.21	0.25	0.23	0.24
AA	0.07	0.07	0.07	0.09	0.09	0.10
A	0.23	0.27	0.24	0.25	0.24	0.26
BBB	0.96	0.96	0.96	0.91	0.89	0.84
BB	2.72	2.75	2.59	2.35	2.28	2.14
B	8.67	6.19	7.61	6.12	5.90	5.25
CCC	24.96	19.40	16.48	13.65	12.68	11.81
Overall	4.43	3.78	3.62	3.18	3.04	2.85

(continued)

Table I. Continued

Panel B1: Firm characteristics by sectors

Sector	Sample firms		Equity volatility (%)	Leverage ratio (%)	Asset payout (%)	Recovery rate (%)
	Number	Percentage				
Communications	6	6.45	48.72	42.93	1.99	40.14
Consumer cyclical	32	34.41	38.95	48.56	2.01	40.45
Consumer staple	14	15.05	33.77	41.68	2.24	40.87
Energy	8	8.60	39.93	53.89	2.47	40.05
Industrial	18	19.35	40.24	53.90	2.01	39.90
Materials	11	11.83	32.85	49.34	2.73	41.35
Technology	4	4.30	45.22	40.20	1.29	38.95
Overall	93	100.00	38.68	48.39	2.14	40.39

Panel B2: Average CDS spreads (%) by CDS maturities and sectors

	Maturity of CDS					
	1-Year	2-Year	3-Year	5-Year	7-Year	10-Year
Communications	2.04	1.99	2.09	2.23	2.16	2.10
Consumer cyclical	1.57	1.58	1.58	1.61	1.62	1.66
Consumer staple	0.74	0.81	0.86	0.92	0.94	0.98
Energy	1.58	1.38	1.53	1.43	1.47	1.48
Industrial	1.29	1.38	1.41	1.46	1.48	1.53
Materials	0.92	0.96	1.03	1.10	1.14	1.20
Technology	1.38	1.43	1.48	1.48	1.51	1.52
Overall	1.34	1.36	1.40	1.44	1.45	1.49

Panel B3: Standard deviation of CDS spreads (%) by CDS maturities and sectors

Communications	4.82	4.13	4.58	4.74	4.33	3.80
Consumer cyclical	6.19	5.25	4.65	4.06	3.85	3.65
Consumer staple	2.08	2.21	2.18	2.10	2.02	1.92
Energy	5.60	3.66	4.80	3.32	3.45	3.14
Industrial	2.36	2.54	2.34	2.16	2.09	2.07
Materials	1.46	1.42	1.43	1.39	1.38	1.34
Technology	2.20	2.17	2.12	1.82	1.74	1.59
Overall	4.43	3.78	3.62	3.18	3.04	2.85

groups, although the BB–CCC group has another spike in late 2004. On the other hand, equity volatility is much higher in 2002 than the later part of the sample period and, in particular, has two huge spikes in 2002. There is clear evidence that equity volatility and credit spreads are intimately related (Campbell and Taksler, 2003), and the linkage appears to be nonlinear in nature (Zhang, Zhou, and Zhu, 2009). In the next section, we examine whether structural models can capture the dynamics of CDS spreads and equity volatility in our sample, among other things.

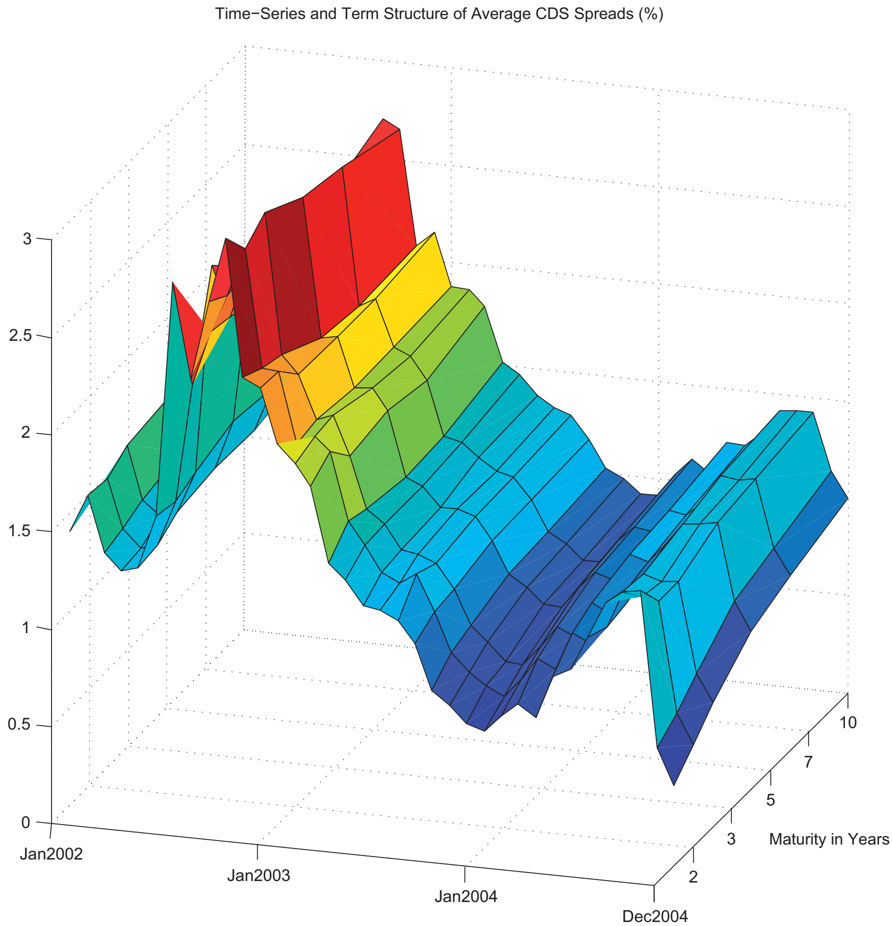

Figure 1. Average CDS spreads over the full sample period. This figure plots the average CDS spreads (in annualized percentage) of 93 firms in the full sample with maturities ranging from 1 to 10 years from January 2002 to December 2004.

6. Empirical Results

In this section, we present the results from our empirical analysis. We consider first the proposed GMM specification test of the five candidate models. We then examine the GMM estimates of model parameters and the pricing performance of the models. We also provide diagnostics on model specifications. Lastly, we focus on the model implications for hedge ratios and default probabilities.

6.1 GMM Specification Test

Our GMM specification test is based on the model-implied pricing relationship for CDS spreads and equity volatility. Table II reports the test results and, in particular, the number of firms where each of the five candidate models is *not* rejected, for the whole sample as well as subsamples by either credit ratings (panel A) or sectors (panel B). Note from the table that at the conservative 10% significance level, the number of firms (out of 93) where

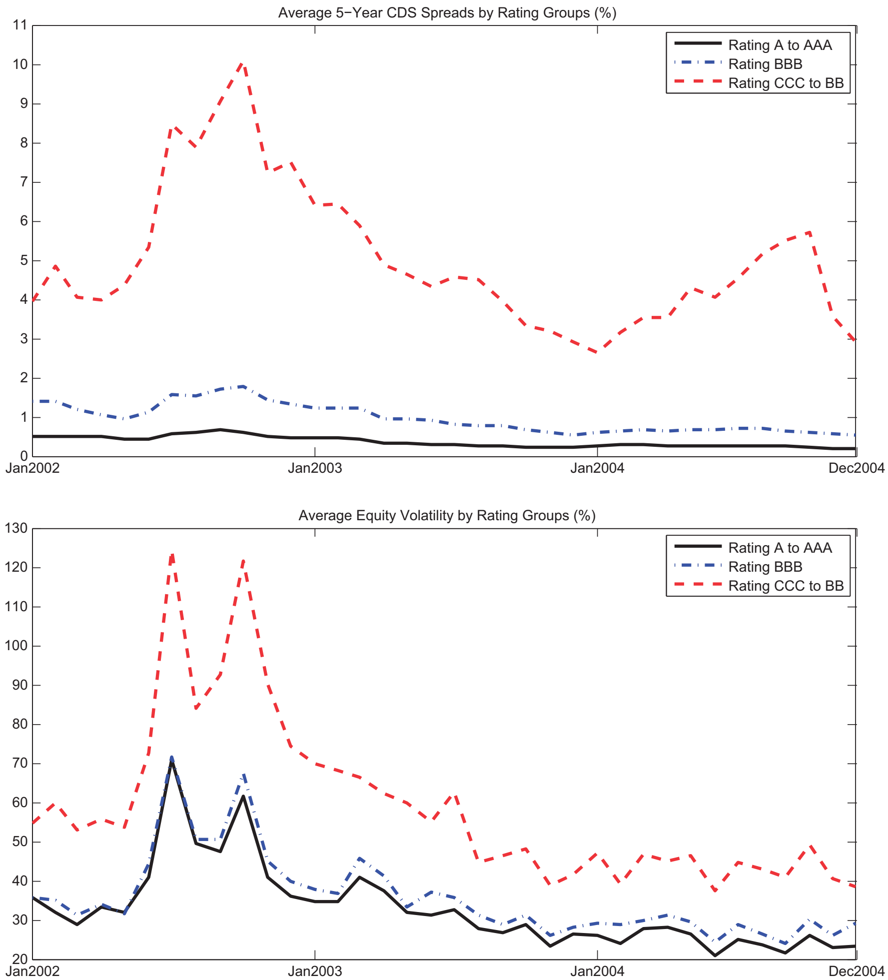

Figure 2. Time series of CDS spreads and equity volatility. This figure plots the average 5-year CDS spread (top panel) and the average realized equity volatility (bottom panel) by rating groups (A–AAA, BBB, and CCC–BB) over the period January 2002–December 2004. Realized equity volatility is estimated using 5-min intraday stock return data.

the given model is not rejected is 0, 1, 2, 13, and 52 for the Merton, BC, LS, DEJD, and CDG models, respectively. At the 1% significance level, none of the five models have a rejection rate of 100% and the number of firms with the model not being rejected increases to 5, 6, 12, 42, and 72 for the Merton, BC, LS, DEJD, and CDG models, respectively. Judged by these results on the number of firms where each of the five models is not rejected, the ranking of these models is as follows:

$$\text{Merton} \approx \text{Black–Cox} < \text{LS} \ll \text{DEJD} < \text{CDG}.$$

Notably, the two more recent models—the DEJD and CDG models—outperform the other three models. This finding implies that both jumps and time-varying leverage improve

Table II. Specification test of structural credit risk models

This table reports the omnibus GMM test results of overidentifying restrictions under each of five structural models considered. The five moment conditions used in the test are constructed based on the pricing relationship for 1-, 3-, 5-, and 10-year CDS spreads and for the equity volatility estimated based on 5-min intraday data. The five model specifications considered include Merton (1974), BC, LS, CDG, and the DEJD model used in Huang and Huang (2002). Data used in the test are monthly CDS spreads and equity volatility from January 2002 to December 2004.

	Number of firms	Structural credit risk models estimated														
		Merton			Black-Cox			Longstaff–Schwartz			DEJD			CDG		
		Chi-square values														
d.o.f.		4			3			3			2			1		
Mean		16.59			14.79			14.41			9.32			3.83		
Percentiles		*p5*	*p50*	*p95*	*p5*	*p50*	*p95*	*p5*	*p50*	*p95*	*p5*	*p50*	*p95*	*p5*	*p50*	*p95*
		12.59	17.12	17.92	12.03	15.08	16.26	13.66	14.53	14.62	1.57	9.36	15.73	0.01	2.21	13.27
Sig. level		0.01	0.05	0.10	0.01	0.05	0.10	0.01	0.05	0.10	0.01	0.05	0.10	0.01	0.05	0.10
Panel A: Number of firms with the model being not rejected: by ratings																
AAA	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0
AA	6	0	0	0	0	0	0	0	0	0	2	1	1	4	4	3
A	25	0	0	0	0	0	0	0	0	0	10	8	6	22	22	21
BBB	44	2	0	0	2	0	0	6	2	1	22	7	4	36	29	23
BB	12	3	0	0	2	1	1	4	2	1	4	1	0	6	4	3
B	4	0	0	0	2	0	0	1	1	0	2	2	1	4	4	2
CCC	1	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0
Panel B: Number of firms with the model being not rejected: by sectors																
Communications	6	1	0	0	2	0	0	1	0	0	3	0	0	5	5	4
Consumer cyclic	32	1	0	0	0	0	0	3	1	0	16	8	4	21	18	15
Consumer stable	14	0	0	0	0	0	0	0	0	0	5	2	2	11	10	7
Energy	8	0	0	0	1	0	0	1	1	0	4	3	1	6	5	4
Industrial	18	1	0	0	1	1	1	4	2	2	9	4	3	16	14	11
Materials	11	1	0	0	1	0	0	2	1	0	5	3	3	10	9	9
Technology	4	1	0	0	1	0	0	1	1	0	0	0	0	3	2	2
Total	93	5	1	0	6	1	1	12	6	2	42	20	13	72	63	52

noticeably the model performance.[7] Although it is known that the Merton model underperforms the richer models, the evidence presented here against the model is based on a consistent econometric test that takes into account the dynamic behavior of both CDS spread curves and equity volatility.

Granted, GMM omnibus tests may be biased toward over-rejection of the true model specification. As a robustness check, we repeat the GMM test of the Merton model using only one CDS contract (the 5-year one) and realized equity volatility. The results from this test with the degree of overidentification being one show that the number of firms with the Merton model not being rejected is still zero at the 10% significance level but increases to 20 at the 1% significance level (untabulated). These results indicate that when the degree of overidentification decreases from four to one, the GMM test indeed rejects the Merton model considerably less at the 1% significance level. Nonetheless, the number of firms not rejecting the model (twenty) is still significantly below that for either the DEJD model (forty-two) or the CDG model (seventy-two).

As such, our findings provide new evidence on the relative performance of the five candidate models. Furthermore, given that even the highest-ranking CDG model is still rejected by twenty-one out of ninety-three firms at the 1% significance level, the results in Table II also indicate that the five representative models considered here are all missing something.

6.2 Parameter Estimation

Although the GMM method provides a consistent test of the models, it does not necessarily force the parameter estimates to be plausible in the estimation. Thus, it is important to examine next the model parameters and model implications for certain moments or variables not included in the moment conditions. We focus on the former task in this subsection and investigate the latter in Sections 6.5 and 6.6.

Recall from Section 4.3 that vector θ does not include those predetermined parameter inputs in the case of the two-factor models or the DEJD model. Table III reports parameter estimates $\hat{\theta}$ and their standard errors across either credit ratings or sectors. Panel A shows the results for the asset volatility parameter σ_v, which enters all five models. This parameter is significant at all conventional statistical levels. The level of the estimates is reasonable in all models: the mean (median) asset volatility ranges from 0.154 (0.135) for the Merton model to 0.199 (0.170) for the CDG model. The standard deviation of $\hat{\sigma}_v$ ranges from 0.007 for the Merton model to 0.09 for the LS model.

Panel B of Table III reports the estimated default barrier scaled by the total debt, an important parameter in the three models with a flat default boundary. The estimated K/F has a mean (median) of 1.18 (1.06), 1.16 (1.05), and 0.83 (0.75) for the BC, LS, and DEJD models, respectively. This result is intuitive albeit not surprising. For instance, relative to the BC model, the LS model needs a slightly lower K due to the interest rate factor with $\rho = 0$ and the DEJD model requires a lower K given the positive impact of the jump risk on the CDS spread.

7 Eom, Helwege, and Huang (2004) find that the CDG model marginally improves the fitting of bond spreads over the LS model. One possible reason why we find that the improvement over LS here is significant is the use of CDS spreads in our tests. Another possible reason is that the risk-neutral leverage parameters are estimated directly here rather than indirectly through their counterparts under $\mathbb{P}$, as alluded in Eom *et al.* (2004).

Table III. Parameter estimation of structural credit risk models

This table reports the GMM estimation results of the model parameters in each of five structural models. The five moment conditions used in the test are constructed based on the pricing relationship for 1-, 2-, 5-, and 10-year CDS spreads and for the equity volatility estimated based on 5-min intraday data. The five model specifications include Merton (1974), BC, LS, CDG, and the DEJD model used in Huang and Huang (2002). Panel A reports the asset volatility parameter estimate σ_v in all five models, Panel B reports the default boundary estimate K in three barrier type models, and Panel C reports jump intensity estimate λ^Q in the DEJD model and dynamic leverage parameters, κ_ℓ, ν, and ϕ, in the CDG model.

Panel A: Estimates of the asset volatility under different structural models

	Number of firms	Structural credit risk models estimated														
Total	93	Merton			Black–Cox			Longstaff–Schwartz			DEJD			CDG		
Mean			0.154			0.168			0.177			0.160			0.199	
Standard deviation			(0.007)			(0.007)			(0.090)			(0.010)			(0.042)	
Percentiles		*p5*	*p50*	*p95*	*p5*	*p50*	*p95*	*p5*	*p50*	*p95*	*p5*	*p50*	*p95*	*p5*	*p50*	*p95*
		0.070	0.135	0.337	0.097	0.151	0.274	0.061	0.153	0.407	0.075	0.154	0.252	0.098	0.170	0.337
Asymptotic SEs		(0.002)	(0.006)	(0.014)	(0.003)	(0.006)	(0.016)	(0.006)	(0.047)	(0.228)	(0.003)	(0.006)	(0.031)	(0.005)	(0.011)	(0.031)
AAA	1	0.133	0.133	0.133	0.100	0.100	0.100	0.160	0.160	0.160	0.141	0.141	0.141	0.114	0.114	0.114
AA	6	0.053	0.075	0.106	0.156	0.243	0.267	0.153	0.291	0.380	0.154	0.186	0.239	0.134	0.242	0.339
A	25	0.068	0.112	0.168	0.090	0.149	0.203	0.088	0.153	0.250	0.128	0.164	0.221	0.114	0.173	0.325
BBB	44	0.095	0.137	0.222	0.102	0.145	0.216	0.058	0.139	0.227	0.072	0.148	0.224	0.077	0.166	0.276
BB	12	0.111	0.198	0.408	0.114	0.194	0.388	0.054	0.205	0.645	0.076	0.135	0.392	0.104	0.206	0.554
B	4	0.263	0.361	0.399	0.183	0.297	0.351	0.072	0.434	0.602	0.075	0.175	0.264	0.105	0.349	0.473
CCC	1	0.369	0.369	0.369	0.270	0.270	0.270	0.054	0.054	0.054	0.054	0.054	0.054	0.042	0.042	0.042
Communications	6	0.069	0.184	0.399	0.113	0.151	0.335	0.074	0.181	0.412	0.138	0.161	0.264	0.107	0.292	0.451
Consumer cyclic	32	0.091	0.150	0.309	0.097	0.167	0.268	0.045	0.153	0.290	0.055	0.150	0.237	0.042	0.167	0.325
Consumer stable	14	0.053	0.081	0.239	0.094	0.146	0.265	0.106	0.148	0.557	0.085	0.159	0.237	0.106	0.171	0.307
Energy	8	0.105	0.134	0.340	0.112	0.158	0.206	0.072	0.150	0.240	0.097	0.142	0.216	0.113	0.161	0.296
Industrial	18	0.097	0.135	0.403	0.091	0.125	0.378	0.066	0.131	0.583	0.086	0.149	0.343	0.095	0.165	0.526
Materials	11	0.081	0.110	0.222	0.107	0.151	0.230	0.108	0.155	0.201	0.101	0.148	0.276	0.115	0.187	0.243
Technology	4	0.098	0.169	0.304	0.176	0.209	0.275	0.198	0.245	0.441	0.174	0.179	0.289	0.233	0.248	0.493

(continued)

Table III. Continued

Panel B: Estimate of the default boundary

	Number of firms	Structural credit risk models considered								
Total	93	Black–Cox			Longstaff–Schwartz			DEJD		
Mean			1.176			1.161			0.830	
Standard deviation			(0.145)			(0.274)			(0.183)	
Percentiles		*p5*	*p50*	*p95*	*p5*	*p50*	*p95*	*p5*	*p50*	*p95*
		0.640	1.055	1.923	0.536	1.049	2.018	0.419	0.752	1.734
Asymptotic SEs		(0.006)	(0.134)	(0.217)	(0.020)	(0.173)	(0.859)	(0.020)	(0.163)	(0.316)
AAA	1	0.971	0.971	0.971	1.086	1.086	1.086	0.723	0.723	0.723
AA	6	0.844	0.905	2.296	0.751	1.034	2.442	0.807	1.553	1.773
A	25	0.887	1.362	2.425	0.751	1.180	2.449	0.304	0.886	1.904
BBB	44	0.638	1.072	1.685	0.597	1.057	1.835	0.443	0.696	1.174
BB	12	0.655	0.867	1.782	0.430	0.921	1.783	0.422	0.672	1.759
B	4	0.550	0.793	0.983	0.333	0.718	1.004	0.380	0.538	0.700
CCC	1	0.959	0.959	0.959	1.011	1.011	1.011	0.706	0.706	0.706
Communications	6	0.612	1.295	1.675	0.449	1.208	1.846	0.087	0.614	1.137
Consumer cyclic	32	0.648	1.094	2.199	0.618	1.174	2.211	0.501	0.749	1.570
Consumer stable	14	0.606	1.007	2.740	0.417	0.894	2.813	0.474	0.822	1.980
Energy	8	0.638	0.938	1.951	0.417	0.925	2.094	0.350	0.656	1.862
Industrial	18	0.660	1.019	1.840	0.635	1.033	1.813	0.405	0.710	0.953
Materials	11	0.865	1.062	1.513	0.756	1.150	1.657	0.427	0.807	1.025
Technology	4	0.655	0.958	1.630	0.548	0.928	1.636	0.635	0.775	1.767

(continued)

Table III. Continued

Panel C: Estimates of other parameters in the DEJD and CDG models													
(1)	(2) Number of firms	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
		Structural credit risk models considered											
Total	93		DEJD						CDG				
Parameter			$\lambda^{\mathbb{Q}}$			κ_{ℓ}			ν			ϕ	
Mean			0.181			15.155			0.222			2.829	
Standard deviation			{0.078}			{3.258}			{0.274}			{2.070}	
Percentiles		*p5*	*p50*	*p95*	*p5*	*p50*	*p95*	*p5*	*p50*	*p95*	*p5*	*p50*	*p95*
		0.042	0.126	0.499	0.446	15.347	35.466	0.069	0.163	1.180	−0.103	1.878	6.242
Asymptotic SEs		(0.009)	(0.029)	(0.132)	(0.007)	(0.062)	(0.439)	(0.003)	(0.008)	(0.137)	(0.048)	(0.181)	(1.867)
AAA	1	0.119	0.119	0.119	15.042	15.042	15.042	0.106	0.106	0.106	1.184	1.184	1.184
AA	6	0.057	0.092	0.227	1.608	17.715	22.097	0.185	0.293	1.988	0.359	3.142	36.208
A	25	0.034	0.113	0.277	10.189	16.826	35.489	0.099	0.173	0.555	1.352	2.279	10.199
BBB	44	0.054	0.123	0.465	8.717	15.357	35.489	0.068	0.142	0.261	−0.095	1.736	2.952
BB	12	0.008	0.209	0.483	0.047	1.414	20.708	−4.117	0.209	1.158	−12.185	1.367	11.763
B	4	0.420	0.493	0.981	0.476	5.191	8.877	0.069	0.261	1.017	−0.797	1.581	6.336
CCC	1	0.580	0.580	0.580	−0.021	−0.021	−0.021	1.566	1.566	1.566	37.416	37.416	37.416
Communications	6	0.044	0.166	0.420	1.905	12.767	15.833	0.191	0.255	1.389	1.428	3.091	29.644
Consumer cyclic	32	0.060	0.151	0.559	0.126	15.434	33.934	0.069	0.164	1.946	−0.099	1.876	32.980
Consumer stable	14	0.043	0.114	0.468	2.982	17.280	35.489	0.073	0.148	0.295	0.148	1.862	3.507
Energy	8	0.040	0.118	0.469	8.877	14.580	19.271	0.090	0.136	0.277	−0.797	1.340	3.508
Industrial	18	0.043	0.107	0.713	0.647	15.437	35.489	0.069	0.146	0.888	−7.485	1.743	2.796
Materials	11	0.057	0.096	0.441	0.062	17.096	24.548	−4.351	0.159	0.277	−0.025	1.908	5.671
Technology	4	0.001	0.088	0.114	0.406	8.633	16.944	0.195	0.418	1.209	−9.763	2.566	12.435

Note also from panel B that the median *K/F* for IG names is higher than the median for HY names across the three aforementioned models. In particular, in the LS model while the median for IG names is greater than one, the median for HY names is below one. Similar results obtain when we plot the estimated *K/F* against the observed leverage ratio F/V_t as in Figure 3: the slope is significantly negative, indicating that a higher *K/F* is associated with a lower observed leverage (which is usually associated with a higher credit rating). These results on a negative relationship between the default boundary and the credit rating/observed leverage are consistent with the evidence documented in Eom, Helwege, and Huang (2004) based on the LS model with corporate bond data.

Columns 3–5 in panel C of Table III report the estimates of the risk-neutral jump intensity parameter (λ^Q) in the DEJD model. Note that the full-sample mean and median of $\hat{\lambda}^Q$ are 0.181 and 0.126, respectively. Across different rating categories, the median $\hat{\lambda}^Q$ levels for HY names are much higher than those for IG names. For instance, the median is 0.123 for BBB names and 0.209 for BB names. This variation in $\hat{\lambda}^Q$ across different rating groups partly explains the negative relation between the estimated default boundary and the credit rating as discussed earlier (panel B of the table).

The remaining columns in panel C of Table III show the estimates of the three leverage parameters in the CDG model, κ_ℓ (Columns 6–8), ν (Columns 9–11), and ϕ (Columns 12–14). Recall that κ_ℓ is the mean-reverting speed of the risk-neutral log leverage ratio $\log(K_t/V_t)$. The full-sample mean and median of $\hat{\kappa}_\ell$ are around 15.16 and 15.35, respectively. In the IG subsample, the median ranges from 15.04 for the single AAA-rated name to 17.72 for the AA-rated names; in the HY subsample, the median is −0.021 for the single CCC-rated name, 1.41 for the BB-rated names, and 5.19 for the B-rated names. These medians of $\hat{\kappa}_\ell$ are much larger than the calibrated value of 0.18 adopted by CDG or the regression-based estimate obtained in Frank and Goyal (2003), regardless of the rating categories except for the single CCC name. This finding indicates that the CDG model may be missing something, thereby illustrating again the importance of post-estimation examination of the parameter estimates.

Parameter ν is related to θ_ℓ, the long-run mean of the risk-neutral leverage ratio, given that $\theta_\ell = \frac{\delta - r_t + \sigma_v^2/2}{\kappa_\ell} + \phi(r_t - \theta_r) - \nu$. Our choice of estimating a constant ν implies a time-varying but deterministic θ_ℓ. The median of $\hat{\nu}$ ranges from 0.11 for the lone AAA name to 1.57 for the only CCC-rated name. The full sample mean and median are 0.22 and 0.16, respectively, both of which are closer to the calibration value of 0.60 used in CDG.

Parameter ϕ measures the sensitivity of the firm-specific leverage ratio dynamics to the risk-free interest rate, similar to the risk factor loading in standard asset pricing models. The full sample mean and median are 2.83 and 1.88, respectively. Across different rating groups, the median lies in between 1.18 and 3.15 except for the single CCC firm whose $\hat{\phi}$ is about 37.42. There is substantial variation in $\hat{\phi}$ within each rating group except for the single AAA- and CCC-rated names. For instance, in the BB group the 5th- and 95th-percentiles are about −12.19 and 11.76, respectively. These results suggest that firms have very different leverage ratio dynamics as the macroeconomic risk changes over time. Such a heterogeneity of leverage ratio dynamics seems to be the key for the CDG model to pass the GMM specification test for more than half of the firms in the sample.

6.3 Pricing Performance Evaluation

Given the estimated (candidate) models, we can also examine their pricing performance. In fact the evaluation of structural models is usually based on comparing their pricing errors on corporate bonds in the literature.

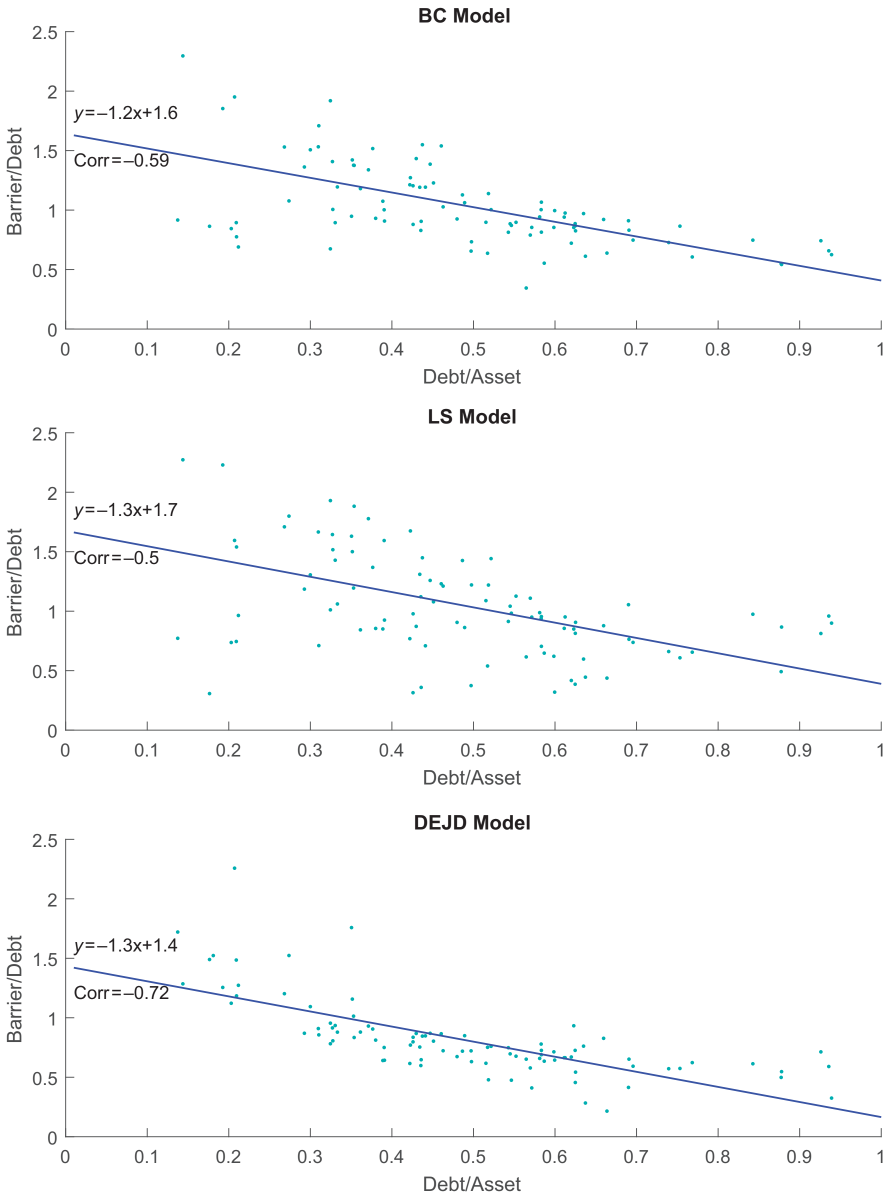

Figure 3. Leverage ratio and the estimated default boundary. This figure shows three scatter plots between the observed leverage ratio (debt/asset) and the estimated default boundary (scaled by debt) for the full sample of 93 firms over the period January 2002–December 2004. The models with a flat default boundary/barrier used in the estimation include BC, LS, and the DEJD model used in Huang and Huang (2002).

To be more specific, given a candidate model and its estimated model parameters, in each month we calculate the model-implied equity volatility and CDS spreads for each maturity including 2 and 7 years. Note that while 2- and 7-year contracts are too sparse to be included in estimation, they are still useful to be included in pricing error evaluation. Then we compute the simple difference, absolute difference, and percentage difference between the model-implied and observed ones, for every name in the sample. Next, we calculate the mean of the pooled pricing errors.

Table IV reports the pricing errors on CDS spreads for the full sample as well as by each rating group and sector. In terms of pricing errors on the spread level (panel A), the overall average pricing error is negative except for the Merton model. This is to say that on average, the Merton model overestimate the CDS spread while the other four models underestimate the spreads.[8] Specifically, the average pricing error is –0.18% for CDG, –0.44% for DEJD, –0.71% for LS, and –0.91% for BC. Thus, the CDG and DEJD models underfit the CDS spread less than the BC and LS models.

Note that the overall positive pricing error of the Merton model is mainly driven by the four B-rated names and the single CCC firm (Delta Air Lines) in the sample. To see that, recall first from Appendix Table AI that these five names all have high leverage and high equity volatility: Delta Air has an equity volatility of 81.9% and a leverage of 93.9%; the average equity volatility and leverage on the four B-rated names are 83.2% and 72.6%, respectively. It is known that the Merton model-implied short-term credit spread with high leverage and equity volatility can be very high (Merton, 1974), consist with our panel C of Figure 4. As a result, the Merton pricing error on these five names is large as reported in panel A of Table IV. Next, note from panel A that the average pricing error for IG names is negative, regardless of the structural models considered; that is, on average, all five candidate models underestimate the CDS spread on IG names, consistent with the findings of Bao (2009) using the BC and DEJD models as well as those of Eom, Helwege, and Huang (2004) and Huang and Huang (2012) based on IG bonds.

In terms of absolute pricing performance (panel B), the BC and LS models outperform the Merton model but underperform the DEJD and CDG models in both the full sample and each of the seven credit-rating groups (except for the single CCC firm where the BC model slightly outperforms CDG). Furthermore, between the two more recent models, the DEJD model performs relatively better for the IG names while the CDG model does better for the HY names (except for the single CCC firm). These results differ from the findings of Eom, Helwege, and Huang (2004) based on corporate bond data that richer model specifications do not necessarily have lower pricing errors.

Results on percentage pricing errors, reported in panel C, indicate that on average, the CDG model overestimates the CDS spread while the other four models underestimate the spread. Among the IG names, the Merton, BS, LS, and DEJD models all underestimate the spread substantially in each of the four rating categories, except that the DEJD model overestimates the single AAA name's spread. On the other hand, the CDG model overestimates the spread for three IG-rated subgroups. These results indicate that although the newer models (DEJD and CDG) do improve upon the older ones (Merton, BC and LS), the CDG model can raise the spread too much for names in certain rating groups in terms of the percentage pricing errors.

8 Predescu (2005) also observes that combining equity price and CDS spreads would make the Merton model overfit the spread.

Table IV. CDS pricing errors in structural credit risk models

This table reports the pricing errors of CDS spreads under each of five structural models. Pricing errors are calculated as the average, absolute, average percentage, and absolute percentage differences between the model implied and observed spreads, across six maturities, 1, 2, 3, 5, 7, and 10 years, and monthly observations from January 2002 to December 2004. The five model specifications include Merton (1974), BC, LS, CDG, and the DEJD model used in Huang and Huang (2012).

Firms by		CDS pricing errors in five different models									
Ratings or Sectors	number	Merton	BC	LS	DEJD	CDG	Merton	BC	LS	DEJD	CDG
		Panel A: Average pricing error (%)					Panel B: Absolute pricing error (%)				
Overall	93	0.37	−0.91	−0.71	−0.44	−0.18	1.54	0.99	0.98	0.75	0.78
AAA	1	−0.14	−0.30	−0.28	0.00	−0.11	0.24	0.30	0.28	0.19	0.20
AA	6	−0.19	−0.12	−0.16	−0.06	−0.03	0.19	0.15	0.16	0.09	0.11
A	25	−0.31	−0.25	−0.25	−0.08	−0.03	0.34	0.28	0.29	0.17	0.18
BBB	44	0.11	−0.63	−0.61	−0.32	0.00	1.23	0.66	0.68	0.45	0.59
BB	12	0.16	−1.65	−1.60	−0.10	−0.07	2.46	1.82	1.95	1.47	1.41
B	4	6.39	−4.34	−4.24	−3.62	−0.89	8.05	4.95	4.68	4.28	3.31
CCC	1	11.55	−12.95	4.00	−8.82	−10.92	17.14	12.96	10.26	9.94	11.43
Communications	6	−0.50	−1.61	−1.84	−1.51	−0.28	1.29	1.62	1.86	1.53	1.08
Consumer cyclic	32	1.02	−1.09	−0.59	−0.50	−0.54	2.27	1.10	1.08	0.72	0.88
Consumer stable	14	0.43	−0.64	−0.67	−0.13	−0.11	1.06	0.67	0.72	0.31	0.34
Energy	8	1.47	−1.04	−0.71	−0.71	−0.20	2.35	1.07	0.78	0.82	0.81
Industrial	18	−0.29	−0.61	−0.53	−0.53	0.08	0.85	0.92	0.90	0.70	0.71
Materials	11	−0.28	−0.69	−0.77	−0.41	0.60	0.81	0.70	0.80	0.54	0.94
Technology	4	−1.17	−1.19	−0.89	1.33	−0.56	1.17	1.19	0.94	1.92	0.97

(continued)

Table IV. Continued

Firms by		CDS pricing errors in five different models									
Ratings or Sectors	number	Merton	BC	LS	DEJD	CDG	Merton	BC	LS	DEJD	CDG
		Panel C: Average percentage pricing error (%)					Panel D: Absolute percentage pricing error (%)				
Overall	93	−29.62	−70.91	−68.94	−11.88	24.42	114.29	76.20	78.17	45.63	68.88
AAA	1	−6.60	−82.04	−74.13	47.96	3.40	70.69	82.04	75.42	80.03	56.99
AA	6	−100.00	−67.72	−85.95	−21.09	−3.91	100.00	82.60	86.11	44.56	69.12
A	25	−82.60	−71.86	−68.88	−1.95	9.69	97.29	78.14	80.27	47.33	55.39
BBB	44	−25.54	−72.85	−69.50	−17.29	40.35	119.78	76.84	78.82	44.37	80.26
BB	12	15.92	−70.77	−69.28	−6.81	16.01	111.45	72.73	76.82	43.01	58.47
B	4	182.82	−50.96	−50.61	−29.71	33.91	196.57	62.82	59.60	51.82	64.48
CCC	1	117.78	−50.82	−7.35	−16.28	−54.47	131.60	50.85	42.83	37.14	58.59
Communications	6	−62.16	−76.83	−77.03	−39.66	31.97	82.63	77.83	79.42	51.39	87.03
Consumer cyclic	32	14.04	−72.77	−72.08	−6.97	6.22	154.04	75.83	80.02	43.77	60.93
Consumer stable	14	−55.69	−71.10	−64.90	−17.18	3.18	107.27	80.71	79.44	38.03	43.72
Energy	8	−5.28	−66.81	−76.71	−23.08	17.52	133.16	71.67	78.17	38.10	50.46
Industrial	18	−51.63	−66.06	−56.16	−19.15	41.55	74.67	76.29	75.49	45.10	71.85
Materials	11	−66.48	−70.35	−73.29	9.67	86.40	85.74	72.67	75.50	58.27	125.59
Technology	4	−87.25	−77.93	−75.78	4.83	−0.78	87.25	79.34	76.47	61.09	60.87

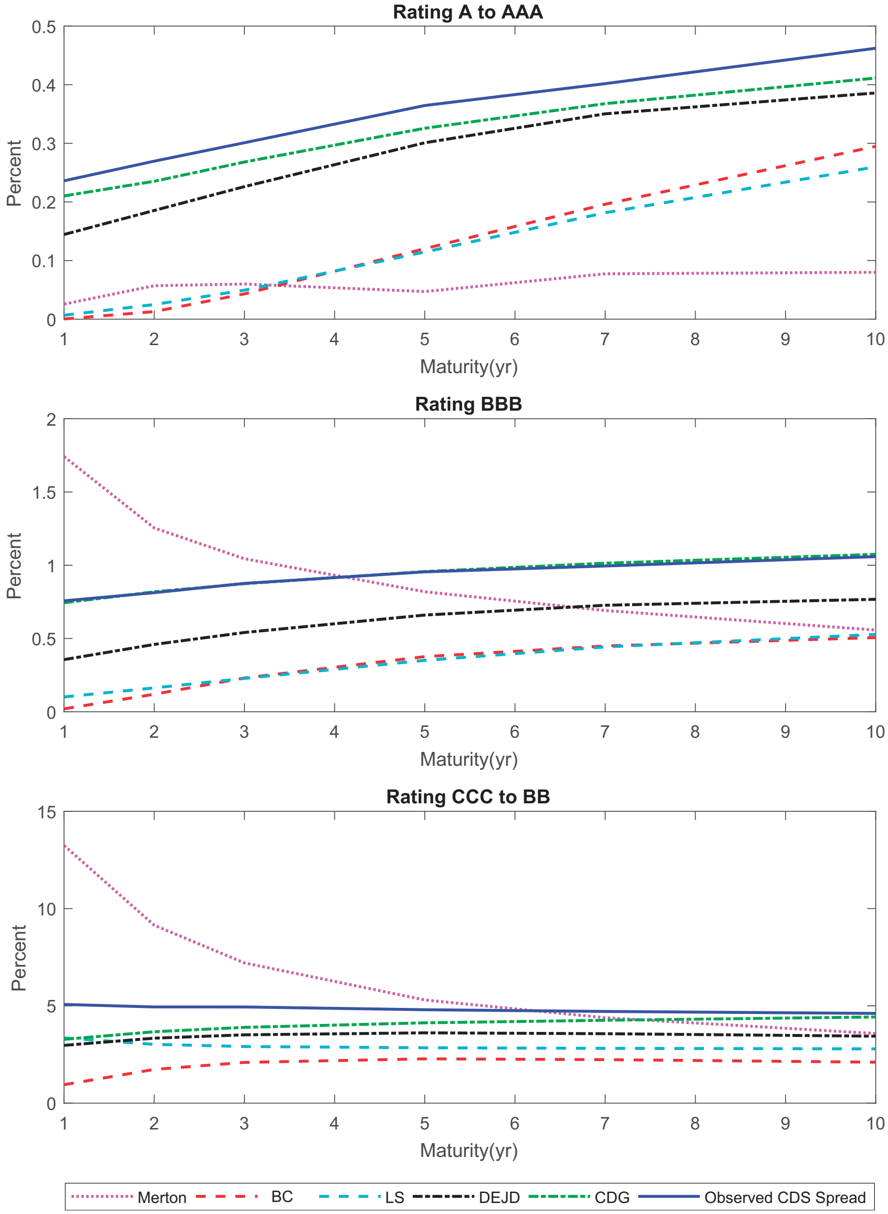

Figure 4. Observed and model-implied CDS term structures. This figure plots the time-series average of both observed and model-implied CDS term structures, by three rating groups, over the period January 2002–December 2004. The structural models considered include Merton (1974), BC, LS, CDG, and the DEJD model used in Huang and Huang (2002).

Panel D reports the results on absolute percentage pricing errors. The ranking of the five models is largely the same as before: the DEJD and CDG models outperform the BC and LS models, both of which outperform the Merton model. Nonetheless, the accuracy of all five models is still a problem: the average absolute percentage pricing error ranges from 45.6% for the DEJD model to 114.3% for the Merton model. This finding echos a similar one in the corporate bond market documented in Eom, Helwege, and Huang (2004).

Table V presents the results on fitting errors of equity volatility. Broadly speaking, they display similar patterns to those on the CDS spreads (Table IV). For instance, consider panel A. Note that for each model the overall sign of fitting errors on equity volatility is consistent with those on CDS spreads, though the magnitude of volatility fitting errors is generally larger. To some extent, this result is not surprising given that credit spreads increase with the asset volatility in the candidate models. Note also that the Merton fitting error is positive overall mainly because of overfitting in the four B-rated and one CCC-rated bonds. In fact, the model underfits equity volatility of AA and A names substantially. The other four models also underfit equity volatility of IG names, except for the single AAA-rated name in the case of the BC, LS, and DEJD models and for the AA-rated names in the case of the BC model.

In terms of absolute fitting performance (panel B), on average, the DEJD and CDG models have the lowest errors (11.61% and 11.87%, respectively), while the Merton model has the highest one (26.11%). The BC model slightly underperforms CDG but outperforms LS substantially. Between the two more recent models, on average, the DEJD model underperforms CDG in IG names but outperforms CDG in HY names.

In terms of percentage fitting errors on equity volatility (panel C), the overall sign is consistent with those on CDS spreads for the BC, DEJD, and CDG models. This is not the case, however, for the Merton and LS models, which both have an overall positive volatility fitting error. Additionally, note that the magnitude of overall percentage fitting errors on equity volatility is much lower than its counterpart on spreads, because the level of equity volatility is typically higher than the CDS spread.

The ranking of the five models based on the overall absolute percentage fitting error on equity volatility (panel D) is the same as that based on the overall absolute fitting error on equity volatility (panel B) except that the BC and CDG models switch their places. In addition, for each of the seven different rating groups, the DEJD model outperforms the CDG model except for the single AAA-rated name.

To summarize, the results of this section provide evidence that the two more recent models (the DEJD and CDG models) outperform the three older ones (the Merton, BC, and LS models) in fitting CDS spreads as well as equity volatility. Nonetheless, we find that on average, the five structural models all underestimate CDS spreads as well as equity volatility for IG names. In addition, the accuracy of all five models in fitting either the CDS spread or equity volatility is low.

6.4 Further Diagnostics on Model Specifications

In this subsection, we examine the average CDS term structure as well as the time series of the 5-year CDS spread and equity volatility. Doing so can provide further insights on model specification errors and consequently on how to improve the models.

Figure 4 plots the sample average of the CDS term structure from 1 to 10 years from the observed data (in solid blue) as well as each of the five candidate models, for three different

Table V. Fitting errors of equity volatility in structural credit risk models

This table reports the fitting errors of equity volatility under each of five structural models. Fitting errors are reported as the average, absolute, average percentage, and absolute percentage differences between the model implied and observed annualized equity volatility, across monthly observations from January 2002 to December 2004. The fitted errors of equity volatility are calculated in a similar fashion. The five model specifications include Merton (1974), BC, LS, CDG, and the DEJD model used in Huang and Huang (2012).

Firms by		Fitting errors of equity volatility in five different models									
Ratings or Sectors	number	Merton	BC	LS	DEJD	CDG	Merton	BC	LS	DEJD	CDG
		Panel A: Average fitting error (%)					Panel B: Absolute fitting error (%)				
Overall	93	5.32	−6.36	−0.77	−7.21	−0.53	26.11	12.09	17.01	11.61	11.87
AAA	1	−3.69	1.03	7.31	1.83	−5.03	12.04	14.42	15.55	13.56	11.81
AA	6	−23.91	−2.29	2.84	−7.97	−0.99	23.91	10.46	13.32	9.91	9.01
A	25	−13.77	−4.95	−6.78	−6.26	−1.15	15.66	10.00	10.66	9.50	8.40
BBB	44	1.92	−5.93	−2.88	−6.56	−1.16	18.32	10.48	13.20	10.01	9.42
BB	12	11.99	−6.82	12.08	−8.81	−1.16	23.79	14.08	24.70	15.89	16.10
B	4	75.80	−31.88	29.71	−17.42	−2.24	83.67	33.62	69.56	28.69	32.70
CCC	1	455.21	15.45	−64.06	−3.47	64.65	455.21	32.11	64.06	23.46	90.07
Communications	6	−8.80	−12.79	−6.90	−16.82	−3.37	18.82	17.07	17.90	18.56	18.03
Consumer cyclic	32	20.28	−5.89	−5.46	−7.32	1.10	40.10	11.72	16.20	11.02	12.46
Consumer stable	14	−1.46	−4.81	10.04	−4.15	−0.90	26.77	11.04	21.15	9.50	9.09
Energy	8	12.43	−13.50	−4.03	−2.80	0.76	26.93	15.37	22.73	10.35	11.94
Industrial	18	−2.68	−4.22	0.17	−8.48	−1.68	13.29	11.34	14.82	12.19	11.05
Materials	11	−7.08	−2.26	1.28	−4.84	−2.92	11.74	9.03	10.30	8.70	9.27
Technology	4	−13.54	−12.47	4.63	−12.17	1.19	18.44	16.37	24.39	21.18	18.43

(continued)

Table V. Continued

Firms by		Fitting errors of equity volatility in five different models									
Ratings or Sectors	number	Merton	BC	LS	DEJD	CDG	Merton	BC	LS	DEJD	CDG
		Panel C: Average PCT fitting error (%)					Panel D: Absolute PCT fitting error (%)				
Overall	93	7.09	−5.44	6.40	−8.96	7.33	58.29	27.88	40.49	25.53	28.41
AAA	1	5.03	21.40	39.47	22.23	0.60	31.98	43.20	51.85	41.33	30.02
AA	6	−72.12	6.57	25.67	−15.04	6.61	72.12	31.67	45.25	25.31	27.39
A	25	−37.18	−4.21	−11.83	−8.99	5.20	44.10	28.02	29.56	24.74	25.49
BBB	44	12.06	−7.42	1.02	−9.32	6.11	48.81	26.10	35.00	24.02	25.59
BB	12	36.98	−4.59	39.25	−6.92	8.54	53.62	27.91	58.90	29.27	34.20
B	4	120.65	−28.54	64.24	−11.47	21.05	125.78	32.33	95.26	31.34	45.24
CCC	1	559.59	33.77	−75.23	−1.53	55.91	559.59	46.11	75.23	29.67	92.76
Communications	6	−13.47	−11.92	−10.51	−21.16	10.67	38.70	29.91	32.49	29.44	38.85
Consumer cyclic	32	37.53	−6.48	−4.13	−11.58	7.48	84.72	27.37	38.63	24.56	27.90
Consumer stable	14	−20.12	−0.98	26.74	−3.49	5.77	67.18	30.26	50.61	25.48	26.17
Energy	8	19.65	−18.90	15.72	−0.47	11.07	54.35	27.63	53.61	24.22	28.89
Industrial	18	−2.65	−0.80	2.79	−9.97	6.66	32.14	27.43	34.52	25.08	25.63
Materials	11	−18.31	0.83	11.02	−5.59	0.41	33.54	25.44	31.32	22.54	25.54
Technology	4	−21.83	−14.32	29.84	−10.53	21.20	38.76	29.83	57.78	40.55	44.02

credit-rating groups, AAA–A (top panel), BBB (middle), and BB–CCC (bottom). A few observations are worth mentioning here: (1) all five models underfit the average term structure except for the Merton model that overfits the short end for the BBB and BB–CCC groups; (2) the best-fitting model, CDG, fits the BBB average term structure almost perfectly and underfits slightly for the AAA–A group; (3) the DEJD model is the second best; (4) the BC model largely captures the shape of the average term structure but underfits its level considerably; (5) the LS model slightly underperforms the BC model for IG names with short maturity but outperforms the latter for HY names; (6) the Merton model underfits the AAA–A curve substantially, especially in the long end but underfits the long end of the BBB and BB–CCC curves less than the BC and LS models.

Overall, both the CDG and DEJD models match the shape of the average term structure of CDS spreads well, especially for IG names. The two models, however, still underfit the level of the curve, although the CDG model-implied curve is much closer to the observed one than the DEJD-implied curve is.

Figure 5 plots the observed 5-year CDS spread against the five model-implied ones. For the HY names (the BB–CCC group), all models seem to capture the time-variations of the 5-year CDS spread reasonably well, although the DEJD and CDG models seem to be the best two. Furthermore, while the DEJD model outperforms the CDG mode in the first third of the sample period, the latter outperforms the former in the last third of the sample period. For the IG names (the AAA–A and BBB groups), most models completely miss the dynamics of the CDS spread, especially for the first third of the sample, when the risk-free rate remains as low as 1%. Interestingly, even the best-fitting CDG model that can get the average level right is not able to describe the evolution of the CDS spread. This finding suggests that a time-varying factor in addition to the interest rate and leverage ratio—like stochastic asset volatility—may be needed in order for a structural model to fully capture the temporal changes in CDS spreads for IG names.

Figure 6 reports the average model-implied and realized equity volatilities over the full sample period, for three different credit-rating groups, AAA–A (top panel), BBB (middle), and BB–CCC (bottom). Note that for both IG groups, all five models miss completely the volatility spikes during the early sample period. Moreover, every model generates a nearly constant equity volatility while the observed equity volatility varies substantially over time. For the HY group, the model performance is relatively better. In particular, the Merton model captures the volatility spikes to some degree and the LS and DEJD models reasonably fit the second half of the volatility time series. However, these results are mainly driven by the unrealistically high model-implied volatility for the single CCC-rated name. Overall, Figure 6 provides evidence suggesting that without time varying asset volatility, the structural models have difficulty replicating the observed equity volatility dynamics, especially for IG names.

Figure 7 plots the initial spot log leverage ratio $\log(K_t/V_t)$ and the long-run mean of risk-neutral log leverage ratio implied from the CDG model, for three different credit-rating groups, AAA–A (top panel), BBB (middle), and BB–CCC (bottom). It is clear from the figure that these two leverages are fairly close to each other for the HY group (the CCC–BB names). On the other hand, for the BBB names the observed leverage is significantly lower than its risk-neutral counterpart, and the difference between the risk-neutral and observed leverages is even more dramatic for the AAA–A names. This finding mirrors the stylized fact that highly profitable firms may opt to borrow little or no debt (Chen and

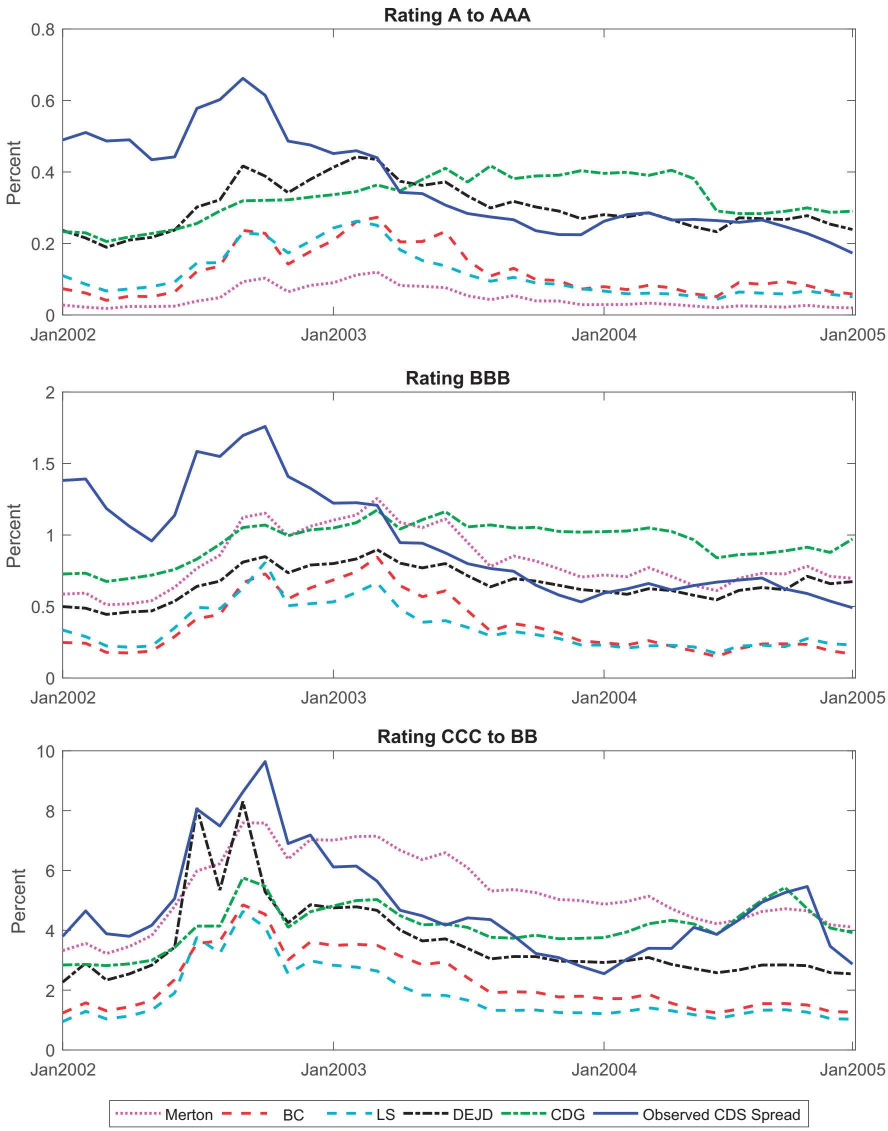

Figure 5. Observed and model-implied 5-year CDS spreads. This figure plots observed and model-implied 5-year CDS spreads, for three credit rating groups, over the period January 2002–December 2004. The structural models considered include Merton (1974), BC, LS, CDG, and the DEJD model used in Huang and Huang (2002).

Zhao, 2006; Strebulaev and Yang, 2013). Such a puzzle may be worth further investigation.

In sum, dynamic leverage ratios and, to a lesser degree, jumps in asset returns help match CDS spreads and equity volatility better. However, something else is still missing in the five candidate models as they all fail to adequately capture the dynamic behavior of CDS spreads and equity volatility, especially for the IG names. Our findings suggest that

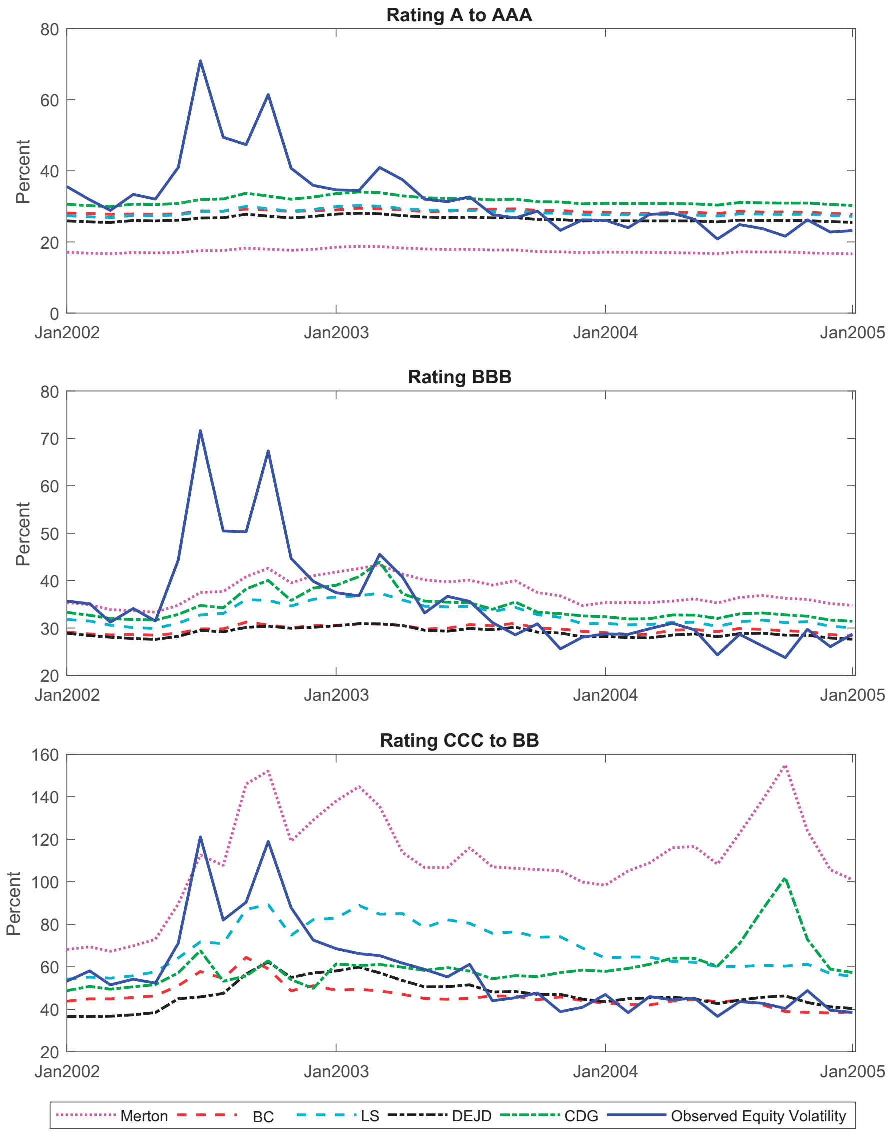

Figure 6. Observed and model-implied equity volatilities. This figure plots realized volatility (estimated using 5-min intraday stock returns) and five model-implied equity volatilities, for three credit rating groups, over the period January 2002–December 2004. The five structural models are Merton (1974), BC, LS, CDG, and the DEJD model used in Huang and Huang (2002).

incorporating a stochastic asset volatility may improve the performance of the existing structural models.

6.5 Model-Implied Equity Sensitivities of CDS Spreads

The implications of the estimated structural models go beyond CDS spreads and equity volatilities, the variables included as moment conditions and examined in Sections 6.3 and 6.4.

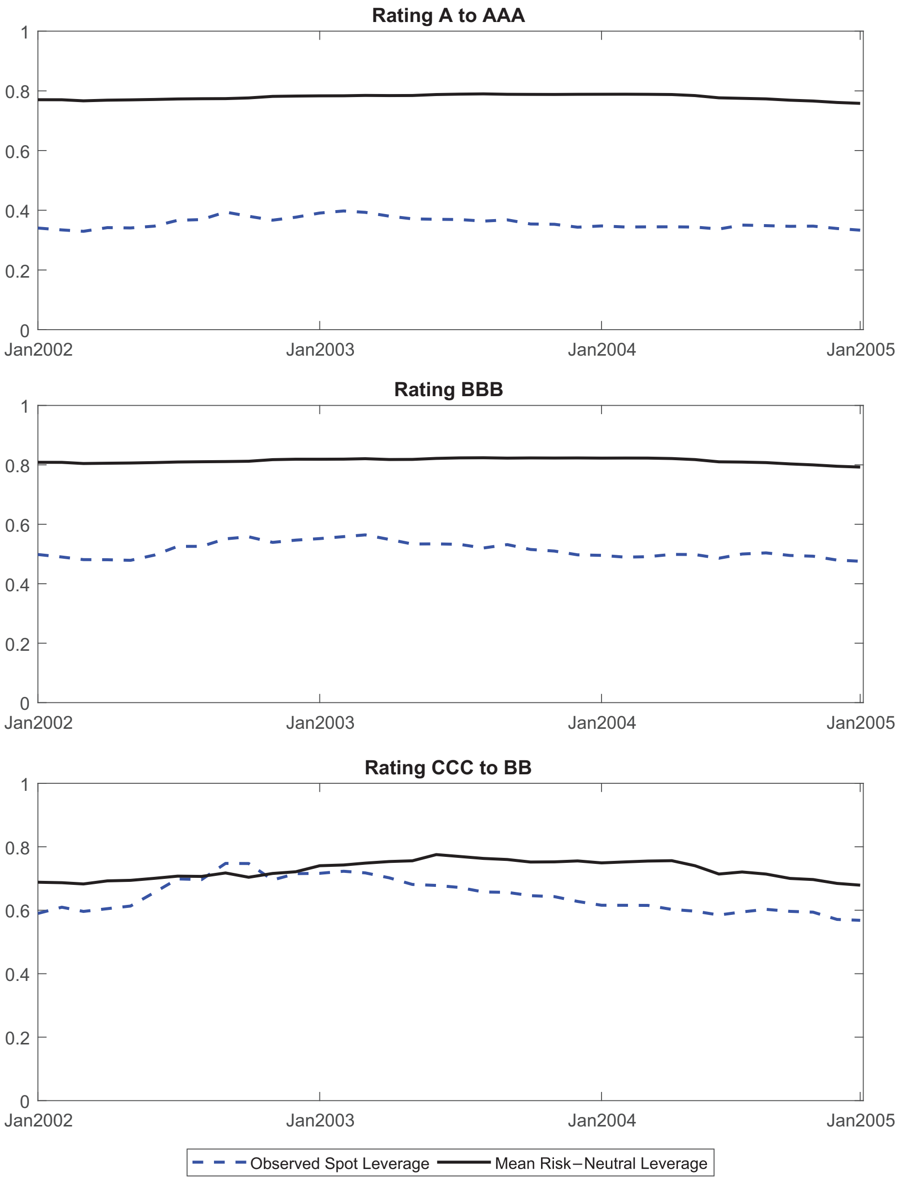

Figure 7. Observed spot leverage and the long-run mean of risk-neutral leverage. This figure plots the observed spot leverage (debt/asset) and the model-implied long-run mean of the risk-neutral leverage, for three rating groups, over the period January 2002–December 2004. The long-run mean of the risk-neutral leverage is estimated using the CDG model.

In this subsection, we focus on one firm-specific variable not included in the moment conditions, the sensitivity of 5-year CDS spreads to equity returns discussed in Section 3.3.

6.5.a. *Regression tests of model-implied sensitivities*

We first test the accuracy of model-implied sensitivities in a linear regression setting. Consider the following regression model:

$$\Delta\widetilde{\text{cds}}(t, t+5)_i = \alpha_i + \beta_{1,i}\Delta r_{f,t}^{10y} + \beta_{2,i}\Delta_{E,i,t}^{\text{cds}} rx_{i,t}^{E} + u_{it}, \tag{19}$$

where $\Delta\widetilde{\text{cds}}(t, t+5)_i$ denotes the monthly change in the observed 5-year CDS spread for firm i; $r_{f,t}^{10y}$ the month-t 10-year zero yield extracted from swap rates and included to control for changes in the "risk-free" term structure; $rx_{i,t}^{E}$ firm-i's monthly equity return minus the one-month LIBOR; and $\Delta_{E,i,t}^{\text{cds}}$ is the model-implied sensitivity of the CDS spread to equity return for firm i as specified in Equation (9)[9] and is calculated using the parameter vector $\hat{\theta}$ estimated with the full sample (see Section 6.2)—for example, $\hat{\theta} = (\hat{\sigma}_v)$ for the Merton model (Section 4.3). If the model accurately describes the equity sensitivity of CDS spreads, $\beta_{2,i}$ should be equal to one. On the other hand, if the model consistently underpredicts the sensitivity, then $\beta_{2,i}$ is expected to be significantly greater than one.

As such, we can test the null hypothesis (H1) that $\beta_{2,i} = 1$ on a firm-by-firm basis and report the number of firms for which H1 is not rejected in our sample. In the analysis that follows, we conduct the test based on a modified Equation (19) with a smoothed $\Delta_{E,i,t}^{\text{cds}}$:

$$\Delta\widetilde{\text{cds}}(t, t+5)_i = \alpha_i + \beta_{1,i}\Delta r_{f,t}^{10y} + \beta_{2,i}\bar{\Delta}_{E,i,t}^{\text{cds}} rx_{i,t}^{E} + u_{it}, \tag{20}$$

where $\bar{\Delta}_{E,i,t}^{\text{cds}}$ denotes the month-t average of model-implied sensitivities across firms in the same rating or industry category as firm i. This is because using a smoothed model-implied hedge ratio can help reduce the noise in the firm-by-firm estimates of model parameters (see, e.g., Schaefer and Strebulaev, 2008).[10]

Table VI reports the results from regression in Equation (20) where $\bar{\Delta}_{E,i,t}^{\text{cds}}$ used is either by ratings (panel A) or by industries (panel B). Consider panel A first. Note that $\bar{\beta}_{2,i}$, the average of the estimates of $\beta_{2,i}$ over the whole sample, is 0.74 and 0.76 for the BC and LS models, respectively, but $\bar{\beta}_{2,i}$ is around one for the other three models. An inspection of the means of $\hat{\beta}_{2,i}$ in each rating category finds that the means are below one regardless of the

9 In the implementation of Equation (9), $\frac{\partial \text{cds}(t,t+5)_i}{\partial V_{i,t}}$ is calculated using Equation (7) and $\frac{\partial E_{i,t}}{\partial V_{i,t}}$ is set to one minus the delta of a 5-year par bond (see footnote 6), an approximation except for the Merton model. In an untabulated analysis using the BC model, we find that including the expected bankruptcy cost in $\frac{\partial E_{i,t}}{\partial V_{i,t}}$ has little impact on the model's performance in fitting both CDS spreads and equity volatility as well as in hedging CDS.

10 The formulation of H1 is in the spirit of Schaefer and Strebulaev (2008) and Huang and Shi (2016), who examine the Merton-implied equity sensitivities of corporate bond returns and spreads, respectively; however, we conduct our hypothesis test slightly differently due to the size of our sample. Those two studies focus on the averages of regression coefficients (counterparts of the $\beta_{2,i}$ estimates here) across bonds in their samples and test whether the mean slope coefficients are close to one. In our case, inferences based on the mean of ninety-three estimates of $\beta_{2,i}$ may not be reliable—given the limited effects of the smoothing in those rating categories or sectors that each include less than ten firms. Still, the means of $\beta_{2,i}$ estimates over the full sample as well as each rating category or sector are reported in Table VI for completeness.

Table VI. Tests of model-implied sensitivities of CDS spreads to equity returns

$$\widetilde{\Delta cds}(t, t+5)_i = \alpha_i + \beta_{1,i}\Delta r_{f,t}^{10y} + \beta_{2,i}\bar{\Delta}_{E,i,t}^{cds} rx_{i,t}^{E} + u_{it},$$

This table reports results from the above time-series regression where $\widetilde{\Delta cds}(t, t+5)_i$ denotes the monthly change in firm-i's 5-year CDS spread; $\Delta r_{f,t}^{10y}$ the monthly change in the 10-year interest rate; $rx_{i,t}^{E}$ the monthly excess returns on firm-i's equity; $\Delta_{E,i,t}^{cds}$ the equity sensitivity of the 5-year CDS spread implied by a given structural model, as given in Equation (9); and $\bar{\Delta}_{E,i,t}^{cds}$ the month-t average of the equity sensitivities across firms in the same credit rating category (panel A) or sector (panel B) as firm i. The five structural models considered include Merton (1974), BC, LS, CDG, and the DEJD model used in Huang and Huang (2012). The reported coefficient values ($\bar{\beta}_{2,i}$) are the averaged estimates of $\beta_{2,i}$ across firms; in angle brackets is reported the number of firms where $\beta_{2,i} = 1$ is not rejected at the 5% significance level; the statistics in brackets are regression R^2s. The sample period is from January 2002 to December 2004.

Regression-related variables	Panel A: Rating-specific average sensitivities							Panel B: Sector-specific average sensitivities						
	Rating	Number of firms	Models used					Sector	Number of firms	Models used				
			Merton	BC	LS	DEJD	CDG			Merton	BC	LS	DEJD	CDG
$\bar{\beta}_{2,i}$	AAA	1	0.92	0.75	0.73	0.87	1.25	Communications	6	1.33	0.70	0.72	1.30	1.25
Number of No-Rej			< 1 >	< 0 >	< 0 >	< 1 >	< 1 >			< 6 >	< 0 >	< 4 >	< 6 >	< 4 >
R^2			[0.307]	[0.580]	[0.532]	[0.286]	[0.269]			[0.421]	[0.101]	[0.185]	[0.418]	[0.185]
$\bar{\beta}_{2,i}$	AA	6	0.70	0.70	0.72	0.70	0.73	Consumer cyclic	32	1.00	0.70	0.73	0.99	1.01
Number of No-Rej			< 0 >	< 0 >	< 1 >	< 0 >	< 4 >			< 25 >	< 0 >	< 7 >	< 26 >	< 26 >
R^2			[0.174]	[0.137]	[0.131]	[0.172]	[0.149]			[0.292]	[0.128]	[0.140]	[0.316]	[0.194]
$\bar{\beta}_{2,i}$	A	25	0.84	0.72	0.74	0.81	0.83	Consumer stable	14	0.81	0.70	0.76	0.81	0.83
Number of No-Rej			< 15 >	< 1 >	< 5 >	< 13 >	< 19 >			< 5 >	< 0 >	< 6 >	< 4 >	< 12 >
R^2			[0.272]	[0.262]	[0.270]	[0.270]	[0.193]			[0.194]	[0.149]	[0.123]	[0.201]	[0.169]

(continued)

Table VI. Continued

Regression-related variables	Panel A: Rating-specific average sensitivities							Panel B: Sector-specific average sensitivities						
	Rating	Number of firms	Models used					Sector	Number of firms	Models used				
			Merton	BC	LS	DEJD	CDG			Merton	BC	LS	DEJD	CDG
$\bar{\beta}_{2,i}$	BBB	44	1.00	0.73	0.76	0.94	1.04	Energy	8	1.16	0.70	0.67	1.23	0.74
Number of No-Rej			< 41 >	< 4 >	< 6 >	< 40 >	< 35 >			< 6 >	< 0 >	< 1 >	< 6 >	< 6 >
R^2			[0.279]	[0.236]	[0.263]	[0.278]	[0.199]			[0.160]	[0.103]	[0.107]	[0.189]	[0.219]
$\bar{\beta}_{2,i}$	BB	12	1.58	0.81	0.76	1.42	0.94	Industrial	18	1.03	0.70	0.68	0.99	1.06
Number of No-Rej			< 10 >	< 3 >	< 2 >	< 10 >	< 12 >			< 15 >	< 0 >	< 5 >	< 13 >	< 16 >
R^2			[0.507]	[0.385]	[0.443]	[0.506]	[0.180]			[0.268]	[0.099]	[0.139]	[0.277]	[0.133]
$\bar{\beta}_{2,i}$	B	4	2.90	0.88	0.83	2.52	1.25	Materials	11	0.92	0.70	0.70	0.90	0.88
Number of No-Rej			< 4 >	< 3 >	< 3 >	< 4 >	< 4 >			< 7 >	< 0 >	< 0 >	< 7 >	< 8 >
R^2			[0.390]	[0.360]	[0.384]	[0.393]	[0.110]			[0.276]	[0.107]	[0.111]	[0.300]	[0.166]
$\bar{\beta}_{2,i}$	CCC	1	3.90	0.95	0.86	3.49	1.00	Technology	4	1.43	0.70	0.69	1.37	0.80
Number of No-Rej			< 1 >	< 1 >	< 1 >	< 1 >	< 1 >			< 4 >	< 0 >	< 2 >	< 3 >	< 1 >
R^2			[0.152]	[0.089]	[0.095]	[0.171]	[0.054]			[0.596]	[0.124]	[0.109]	[0.669]	[0.423]
$\bar{\beta}_{2,i}$	Overall	93	1.12	0.74	0.76	1.05	0.96	Overall	93	1.02	0.70	0.71	1.01	0.96
Number of No-Rej			< 72 >	< 12 >	< 18 >	< 69 >	< 76 >			< 68 >	< 0 >	< 25 >	< 65 >	< 73 >
R^2			[0.304]	[0.263]	[0.286]	[0.302]	[0.187]			[0.281]	[0.119]	[0.132]	[0.300]	[0.186]

rating categories for both the BC and LS models. This result indicates that these two models consistently overpredict the equity sensitivity of CDS spreads. On the other hand, for the Merton and DEJD models, the average $\hat{\beta}_{2,i}$ is below or very close to one for IG names but is greater than one for HY names—and, in fact, the pair of the coefficients for B and CCC names are (2.90, 3.90) and (2.52, 3.49) for the Merton and DEJD models, respectively. The variation in the average $\beta_{2,i}$ across different rating categories is much less for the CDG model, with the average $\beta_{2,i}$ ranging from 0.73 for AA names to 1.25 for AAA- or B-rated names.

For how many firms out of 93 the null hypothesis H1 is not rejected (for a given model), based on the *t*-statistics using the Newey–West standard error estimator? As indicated in panel A, the answer is 72 (Merton), 12 (BC), 18 (LS), 69 (DEJD), and 76 (CDG), at the 5% significance level. Recall from Table II that the number of firms where a given model is not rejected by the GMM-based specification test at the 5% significance level is 1 (Merton), 1 (BC), 6 (LS), 20 (DEJD), and 63 (CDG). That is, all five models capture the sensitivity of CDS spreads to equity much better than they capture CDS spreads and equity volatility. This is true especially for the Merton model.

Regression R^2, shown in the last row of panel A, is 30.4% for Merton, 26.3% for BC, 28.6% for LS, 30.2% for DEJD, and 18.7% for CDG. Note that the R^2 generated by the CDG model is the lowest among the five models—and even lower than its counterpart from the otherwise same regression excluding $\bar{\Delta}^{\mathrm{cds}}_{E,i,t}$ (untabulated). How to reconcile this result with the evidence that the number of firms where H1 is not rejected is the highest under CDG? One explanation is that the *t*-test conducted at the firm level may fail to reject the null hypothesis even if the point estimate of the slope coefficient substantially deviates from unity, due to the large standard error estimated using the Newey–West adjustment. Therefore, although among the five candidate models the CDG model has the largest number of non-rejected firms, the model does not necessarily make the most accurate prediction of hedge ratios.

The results reported in panel B of Table VI are largely similar to those in panel A. For example, the means of estimated $\beta_{2,i}$ in every sector are 0.70 for the BC model and below 0.76 for LS. On the other hand, the means are much closer to one for the other three models. Furthermore, the Merton-based mean estimate is the largest among the five model-based mean estimates for three sectors (out of seven), including 1.33 for "communication," 0.92 for "materials," and 1.43 for "technology," and the second largest for the remaining four sectors. In terms of the regression R^2, it is 28.1% for Merton, 11.9% for BC, 13.2% for LS, 30.0% for DEJD, and 18.6% for CDG. Note that although the R^2 under CDG is not the lowest here, it is still much lower than the R^2-value under either Merton or DEJD.

To summarize, while the results of the test of Hypothesis H1 favor the DEJD, Merton, and CDG models (in ascending order), the first two rank notably higher than CDG based on the regression R^2. As a low R^2-value suggests that the underlying model has difficulty in replicating the variation in CDS contract values effectively, the actual hedging performance of the same model may also be affected negatively. As such, the Merton and DEJD models may provide better hedging performance than does the CDG model. Furthermore, given that the Merton-implied sensitivity is more reasonable than the DEJD-implied one (e.g., for B and CCC names), the Merton model may provide better hedging performance than the DEJD model. In the subsection that follows we investigate which of the five candidate models delivers the most robust hedging performance.

6.5.b. Evidence on hedging effectiveness

Suppose that in month t, an investor hedges a single-name CDS with the underlying equity and makes no additional trades until the end of $t+1$.[11] At $t+1$, the position is closed out and the hedging error over the 1-month period is computed as

$$\epsilon_t = V_{t+1}^{\text{cds}} - h_{E,t}^{\text{cds}} r_{t+1}^{E},$$

where the hedge ratio $h_{E,t}^{\text{cds}}$ is as defined in Equation (10), and we make use of the fact that a CDS contract is worth close to zero when it is first initiated ($V_t^{\text{cds}} = 0$).

Assume that the investor's objective is to minimize the monthly volatility of the hedged single-name CDS. Following Bertsimas, Kogan, and Lo (2000), we use root-mean-squared hedging error (RMSE) as the summary statistic for hedging errors over our sample period. Note that the RMSE is equal to the standard deviation when the mean hedging error is zero. Let $\text{RMSE}_h^{\mathcal{M}}$ be the RMSE when model $\mathcal{M}$-implied hedge ratios are used. For comparison, we also compute the RMSE of the short CDS position when the CDS contract is not hedged ($h_{E,t}^{\text{cds}} = 0$). Denote this RMSE by RMSE_u. One measure of hedging effectiveness calculates the reduction in the RMSE as a result of hedging and is given by

$$H_{\text{Eff}} = 1 - \frac{\text{RMSE}_h^{\mathcal{M}}}{\text{RMSE}_u}. \tag{21}$$

Note that if hedge ratios implied from a particular model substantially increase volatility relative to the unhedged position, then H_{Eff} is negative.

Panel A of Table VII presents the results on the hedging performance of firm-specific hedge ratios (i.e., hedge ratios not smoothed over a given rating group or sector) under each of the five candidate models. Surprisingly, among these models the Merton H_{Eff} is the highest (7.0%), indicating that the Merton-implied hedge ratio achieves the largest reduction in the RMSE. The CDG model also has a significantly positive overall H_{Eff} (3.5%). In contrast, the overall H_{Eff} is highly negative for both the BC and LS models, implying that the hedged position—using hedge ratios derived from the two models—is much more volatile than the unhedged position. The overall negative H_{Eff} for the DEJD model has a great deal to do with the BB-rated names in the sample.

Consider next the hedging performance of the Merton and CDG models by credit ratings or sectors. Note that the Merton H_{Eff} is significantly positive for BB and B names only and that the CDG H_{Eff} is significantly positive for BB names only. On the other hand, out of the seven different sectors, the Merton H_{Eff} is significantly positive for six of them and the CDG H_{Eff} for two. These results together indicate that the Merton hedge ratio is more effective by sectors than by credit ratings.

Why is the overall H_{Eff} so negative for the BC and LS models? One possible reason is that the use of unsmoothed hedge ratios leads to dramatic increases in volatility. Indeed, we observe from Table VI that for those rating or sector groups with a larger number of firms, the (rating- or sector-specific) average hedge ratios tend to be more aligned with their empirical counterparts. This result suggests that smoothing within a credit rating or industry

11 In an untabulated analysis, we also examine the performance of hedging CDS portfolio positions, with the portfolios formed based on the rating/sector category. These results are not reported as the relative performance among structural models does not change; as expected, the absolute hedging effectiveness increases because the hedging loss from one single name in the portfolio may be offset by the hedging gain from another.

Table VII. Hedging performance of structural credit risk models

This table reports empirical results on the effectiveness of hedging changes in CDS spreads with three types of hedge ratios. The first type (panel A) is firm-specific hedge ratios implied from five estimated structural models: Merton (1974), BC, LS, CDG, and the DEJD model used in Huang and Huang (2012). The other two types of hedge ratios are obtained by averaging firm-specific hedge ratios within either each credit rating category (panel B) or each sector (panel C). Given a structural model $\mathcal{M}$, the measure of hedging effectiveness used is $H_{\text{Eff}} = 1 - \text{RMSE}_h^{\mathcal{M}}/\text{RMSE}_u$ as defined in Equation (21), where $\text{RMSE}_h^{\mathcal{M}}$ (RMSE_u) is the root mean square error of the hedged (unhedged) position. The statistics in parenthesis are standard errors of this effectiveness obtained from 5,000 bootstrap simulations. The sample period is from January 2002 to December 2004.

Rating or sector	Number of firms	Hedging performance (H_{Eff}) of various structural models									
		Merton	BC	LS	DEJD	CDG	Merton	BC	LS	DEJD	CDG
		Panel A: Firm-specific hedge ratios					Panel B: Rating-specific average hedge ratios				
AAA	1	−0.817	0.088	0.175	0.101	0.009	−0.817	0.088	0.175	0.101	0.009
		(0.176)	(0.833)	(0.632)	(0.085)	(0.177)	(0.176)	(0.833)	(0.632)	(0.085)	(0.177)
AA	6	0.001	−0.327	0.046	−0.095	0.018	0.030	−0.024	0.050	−0.069	0.035
		(0.050)	(0.318)	(0.251)	(0.033)	(0.070)	(0.000)	(0.035)	(0.023)	(0.041)	(0.022)
A	25	−0.067	−1.684	0.052	−0.129	0.014	0.099	−0.615	0.118	0.115	0.043
		(0.038)	(0.155)	(0.119)	(0.016)	(0.034)	(0.021)	(0.141)	(0.022)	(0.030)	(0.009)
BBB	44	0.005	−90.298	−113.276	0.001	0.022	0.089	−1.818	−2.057	0.109	0.053
		(0.018)	(0.116)	(0.093)	(0.012)	(0.027)	(0.022)	(1.154)	(1.147)	(0.022)	(0.032)
BB	12	0.113	−2.898	−2.237	−21.019	0.184	0.260	−0.058	0.083	−1.822	0.163
		(0.042)	(0.223)	(0.176)	(0.023)	(0.050)	(0.109)	(0.092)	(0.101)	(0.543)	(0.089)
B	4	0.119	−30.461	−0.114	0.039	0.045	0.112	−13.806	0.014	0.073	0.062
		(0.064)	(0.410)	(0.308)	(0.041)	(0.088)	(0.063)	(6.826)	(0.047)	(0.022)	(0.118)
CCC	1	0.058	−9.933	−0.053	0.018	0.001	0.058	−9.933	−0.053	0.018	0.001
		(0.307)	(0.806)	(0.639)	(0.085)	(0.173)	(0.307)	(0.806)	(0.639)	(0.085)	(0.173)
Overall	93	0.070	−29.824	−23.849	−5.133	0.035	0.099	−11.389	−0.359	−0.186	0.058
		(0.016)	(0.080)	(0.053)	(0.008)	(0.018)	(0.041)	(3.628)	(0.215)	(0.112)	(0.040)

(continued)

Table VII. Continued

Rating or sector	Number of firms	Hedging performance (H_{Eff}) of various structural models									
		Merton	BC	LS	DEJD	CDG	Merton	BC	LS	DEJD	CDG
							Panel C: Sector-specific average hedge ratios				
Communications	6	0.143	0.098	0.133	0.082	0.090	0.124	0.104	0.078	0.067	0.050
		(0.064)	(0.322)	(0.250)	(0.033)	(0.071)	(0.027)	(0.063)	(0.038)	(0.017)	(0.060)
Consumer cyclic	32	0.009	−9.674	−0.011	0.017	0.004	0.050	−0.011	0.039	0.047	0.032
		(0.027)	(0.137)	(0.107)	(0.014)	(0.031)	(0.009)	(0.167)	(0.007)	(0.008)	(0.005)
Consumer stable	14	−0.054	0.003	0.030	0.060	0.193	0.060	0.053	0.056	0.057	0.052
		(0.039)	(0.208)	(0.158)	(0.021)	(0.047)	(0.015)	(0.030)	(0.010)	(0.014)	(0.023)
Energy	8	0.096	−36.861	−0.134	0.015	0.012	0.082	−5.429	0.028	0.048	0.030
		(0.056)	(0.267)	(0.214)	(0.028)	(0.061)	(0.036)	(5.189)	(0.014)	(0.024)	(0.016)
Industrial	18	−0.056	−71.679	−103.578	−2.329	0.206	0.113	−6.520	−7.482	0.207	0.096
		(0.040)	(0.186)	(0.150)	(0.020)	(0.041)	(0.097)	(2.249)	(2.243)	(0.092)	(0.057)
Materials	11	−0.297	−4.101	0.035	0.159	−0.198	0.045	−0.055	0.181	0.146	0.107
		(0.065)	(0.228)	(0.187)	(0.024)	(0.053)	(0.069)	(0.088)	(0.087)	(0.051)	(0.037)
Technology	4	0.148	0.098	−4.252	−38.955	0.076	0.208	0.171	−0.241	−12.060	0.075
		(0.150)	(0.395)	(0.317)	(0.042)	(0.087)	(0.070)	(0.066)	(0.506)	(3.304)	(0.028)
Overall	93	0.070	−29.824	−23.849	−5.133	0.035	0.076	−3.310	−1.206	−1.179	0.042
		(0.016)	(0.080)	(0.053)	(0.008)	(0.018)	(0.018)	(1.331)	(0.738)	(0.571)	(0.023)

group could lower the impact of uncertainty in the firm-by-firm estimation, as advocated by Schaefer and Strebulaev (2008). As such, using smoothed hedge ratios (i.e., either rating- or sector-specific (average) hedge ratios) should help mitigate this so-called "hedging crash risk."

Panel B of Table VII reports the results on hedging performance of rating-specific hedge ratios. A comparison with panel A of the table indicates that the overall H_{Eff} in panel B is much less negative for the BC, LS, and DEJD models and, in fact, becomes statistically insignificant for the latter two models.[12] Although CDG's overall H_{Eff} also increases from 3.5% to 5.8%, it is not significantly different from zero. On the other hand, the Merton overall H_{Eff} increases from 7.0% to 9.9% and remains highly significant.

The hedging performance in individual rating groups also improves. For instance, the Merton H_{Eff} is now significantly positive for five out of seven groups (only two out of seven in panel A). For the BC model, its H_{Eff} for the BBB group, for example, increases from −90.3 (highly significant) in panel A to −1.82 (no longer significant) in panel B. For the LS model, its H_{Eff} for the BBB group also increases from a highly significant −113.3 in panel A to an insignificant −2.06 in panel B.

Results on hedging performance of sector-specific average hedge ratios, reported in panel C of Table VII, provide similar evidence as those in panel B do. Consider the overall H_{Eff} first. Note that again, H_{Eff} is much less negative for the BC, LS, and DEJD models than its counterparts in panel A, although it is still significant for the BC and DEJD models.[13] The CDG H_{Eff} is more positive and still significantly different from zero. The Merton H_{Eff} also increases slightly and remains highly significant. Overall, judging from the whole sample, averaging hedge ratios by ratings is more effective than averaging by industry in improving the hedging performance.

Next, consider H_{Eff} for individual sectors. For example, the LS H_{Eff} for "industrial" increases from −103.6 in panel A to −7.48 (albeit still significant) in panel C. The CDG H_{Eff} is now significantly positive for five sectors, as opposed to two sectors in panel A.

In summary, the results based on both the full sample and rating- or sector-specific subsamples in Table VII provide strong evidence that using smoothed hedge ratios helps improve the hedging performance. Furthermore, based on the hedging performance, the top three ranked models are the Merton, CDG, and DEJD models.

We should note that while the analysis of hedging effectiveness presented here corresponds to an out-of-sample test of hedge ratios, the estimates of model parameters make use of the full sample. In an untabulated analysis, we examine the hedging performance for 2- and 7-year CDS contracts (which are not included in the GMM estimation) and find that the results are consistent with those using the 5-year CDS. In particular, the ranking of the

12 Why is the BC overall H_{Eff} still large and negative with smoothed hedge ratios? The reason is that the BC model-implied hedge ratios are striking for certain firms in the sample. In an untabulated analysis, we find that these firms have an estimated default boundary K/F ranging from 1.26 to 1.54. When the asset value is close to this artificial boundary, the equity value becomes insensitive to the asset value. A low $\partial E_t/\partial A_t$ inflates the model-implied equity sensitivity of the CDS spread.

13 The overall negative H_{Eff} for the DEJD model is mainly caused by a BB-rated technology firm. When this firm is excluded from the sample, the hedging performance of the DEJD model is generally comparable to that of the CDG model (untabulated).

five models based on the their hedging performance remains the same. That is, our findings are robust to the aforementioned look-ahead bias.

6.6 Model-Implied Default Probabilities

The discussion so far has focused on the implications of structural models for variables under the risk-neutral measure. In this subsection, we examine model-implied $\mathbb{P}$-measure default probabilities. For comparison, we also include model-implied default probabilities under the (risk-neutral) $\mathbb{Q}$-measure.

As an important determinant of CDS spreads, risk-neutral default probabilities are straightforward to calculate using an estimated model. In order to calculate real default probabilities, we need to specify the dynamics of the underlying variables under the $\mathbb{P}$-measure and then estimate those $\mathbb{P}$-measure parameters. The GMM-based estimation of such parameters, however, requires that $\mathbb{P}$-measure moment conditions be specified. We do not pursue this approach in this analysis. Instead, we calibrate the $\mathbb{P}$-measure parameters in the analysis that follows when it is necessary.

For illustration we focus on the BC model—the simplest one among the three candidate models with a flat default boundary/barrier—in the analysis that follows. Given the specification of the BC model under $\mathbb{Q}$, its specification under $\mathbb{P}$ involves only one extra parameter, the asset risk premium $\pi_v \equiv \mu_v - r$, where μ_v is the expected asset growth rate. We calibrate π_v using the formula, $\mu_v - r = \sigma_v \times \mathrm{SR}_v$, where SR_v denotes the asset Sharpe ratio (equal to the equity Sharpe ratio under the model). To this end, we set SR_v to 0.23, the equity Sharpe ratio of a median firm according to Chen, Collin-Dufresne, and Goldstein (2008), and then use firm-specific asset volatilities estimated earlier in Section 6.2 to calibrate firm-specific asset risk premiums.

Figure 8 plots the time series and term structure of the BC model-implied default probabilities under either the $\mathbb{Q}$ measure (panel A) or the $\mathbb{P}$ measure (panel B) over the full sample period. A comparison of panel A and Figure 1 indicates that the BC model fails to capture the surface of CDS spreads, given that the model assumes a constant recovery rate. As expected, the default probabilities under $\mathbb{Q}$ are markedly higher than their counterparts under $\mathbb{P}$. Nonetheless, both panels show a spike in late 2002, consistent with Figure 1.

We can also compare the average model-implied real default probability with the average (historical) default rate for a given rating group. For the latter, we use the average issuer-weighted cumulative default rates by rating categories over 1920–2004 calculated by Moody's. Figure 9 plots the term structures of average default rates (solid line), the BC model-implied default probabilities under the $\mathbb{Q}$ measure (blue dashed line) as well as the $\mathbb{P}$ measure (red dotted line), for three different rating groups, single A (panel A), BBB (panel B), and BB (panel C). The AAA–A group is not considered here because, first, we do not have Moody's average default rates for the AAA–A group and secondly, the AAA–A group in our sample is dominated by the single A firms. Panel C includes only the BB names instead of the CCC–BB group for the similar reason.

We make two observations from Figure 9. First, the BC model fits the Moody's average default rates well for A-rated names. The implication of this result is that the evidence based on single A firms in our sample is consistent with the notion of the credit spread puzzle: the model matches the average default rates but it underpredicts the CDS spreads. Second, the model underfits the average default rates for both BBB and BB names, especially at long horizons. To some extent, this result is not surprising given that on average, the model noticeably underestimates the CDS spreads for BBB and BB names over the full

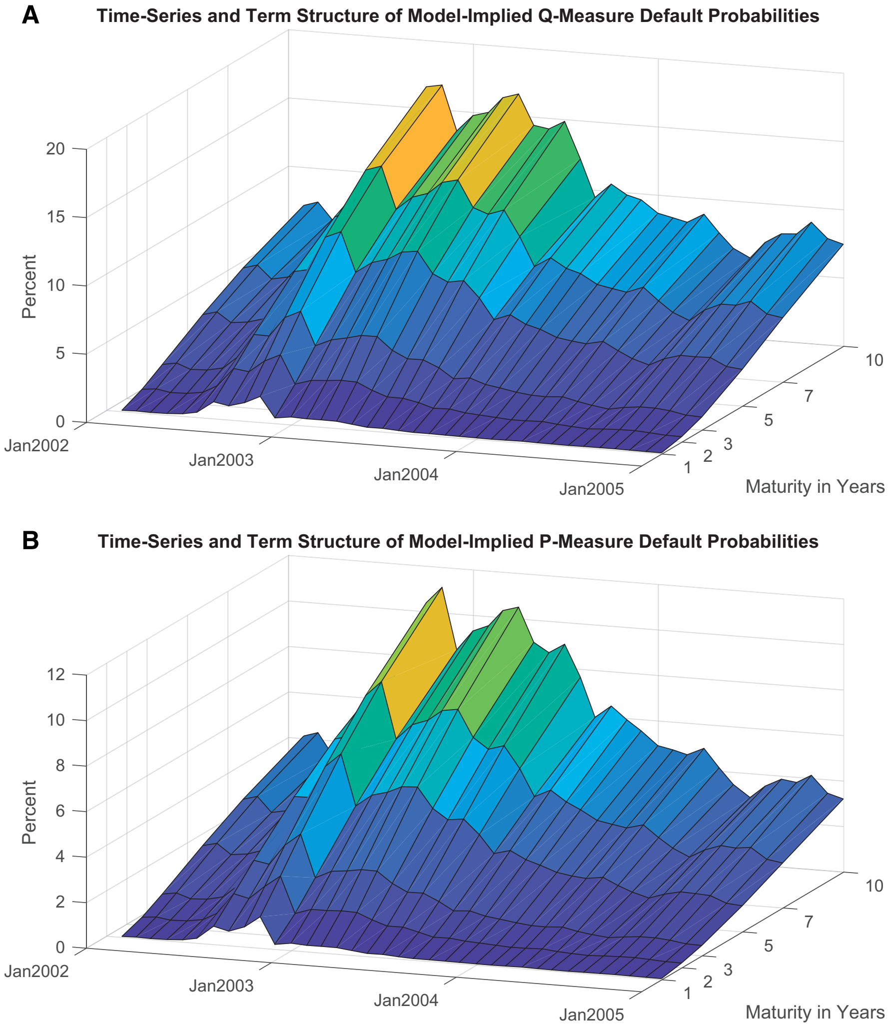

Figure 8. Model implied risk-neutral and real default probabilities. This figure plots the time series and term structure of model-implied default probabilities under either the risk-neutral measure (panel **A**) or the physical measure (panel **B**) based on the BC model.

sample. For the model to match the historical averages the period 1920–2004, we need higher asset volatility, default boundary, or both (than the estimates reported in Table III). Such parameter values also allow the model to fit the observed CDS spreads for BBB and BB names better, largely consistent with the credit spread puzzle.

7. Conclusion

Empirical studies of structural credit risk models are usually carried out using calibration, rolling window estimation, or regression analysis. This paper proposes a GMM-based specification test of these models. This alternative method allows us to directly estimate

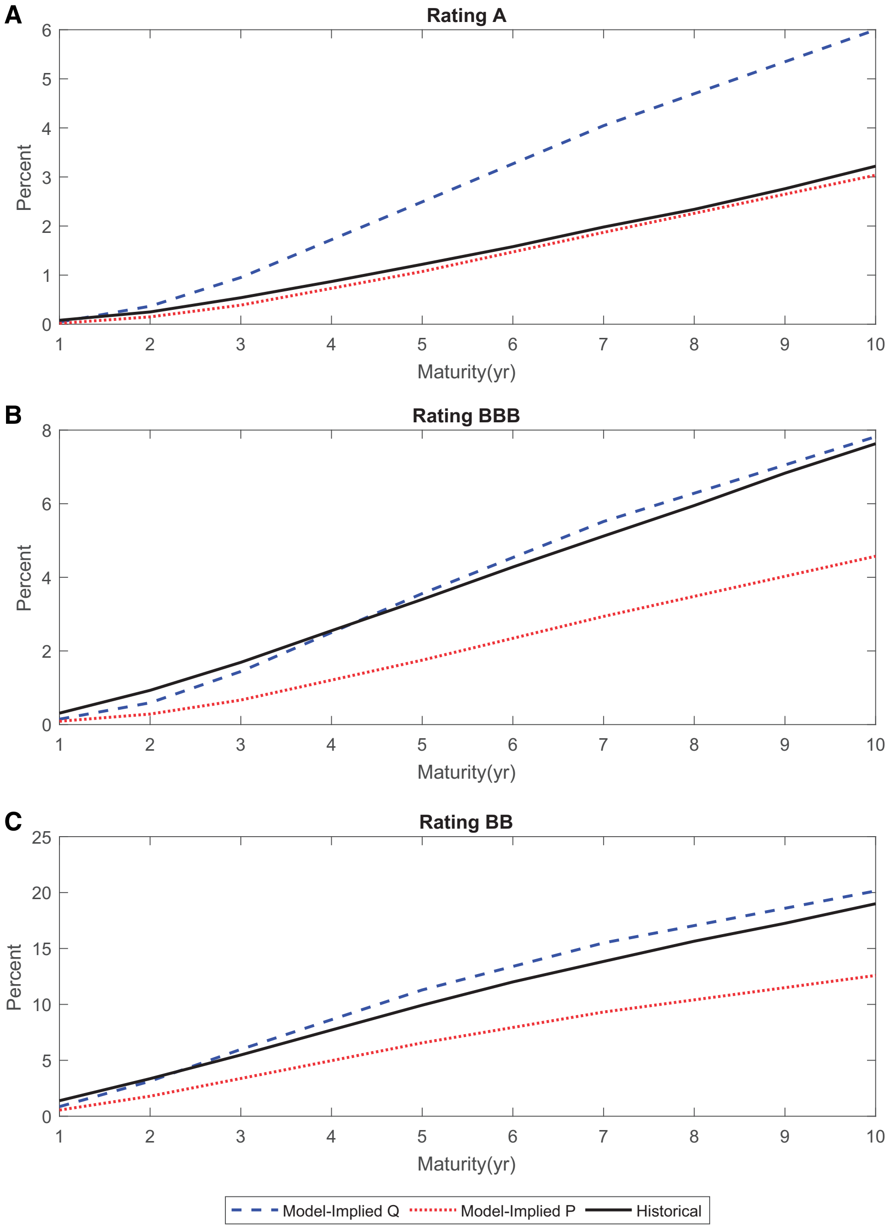

Figure 9. Term structures of average default rates, and model implied risk-neutral and real default probabilities. This figure plots the term structure of average default rates (solid line), model-implied default probabilities under both the risk-neutral measure (blue dashed line), and the physical measure (red dotted line) based on the BC model, for three different rating groups, single A (panel **A**), BBB (panel **B**), and BB (panel **C**).

structural models, as well as test whether all the restrictions of a given model are satisfied, among other things.

For illustration, we apply the proposed specification test to five representative structural models using data on the term structure of CDS spreads and realized equity volatility (estimated with high frequency intraday data). We conduct the test using a sample of industrial firms over a post dot-com bubble and pre-financial crisis period that nonetheless includes some relatively high credit risk episodes. The test results show that the Merton (1974) model and the two diffusion-based constant-barrier models are all strongly rejected by the proposed specification test. However, the results also indicate that incorporating jumps or stationary leverage into a barrier model improves the overall fit of CDS spreads and equity volatility. Nonetheless, all five models have difficulty capturing the dynamic behavior of both equity volatility and CDS spread curves, especially for IG names. On the other hand, our results demonstrate that these models have a much better ability to explain the average sensitivity of CDS spreads to equity returns than their ability to explain the average CDS spread and equity volatility. Surprisingly, we also find that the Merton (1974) model provides the best hedging performance among all five models.

Overall, the main findings of this study, together with those of Bao and Pan (2013) on excess corporate bond return volatility, suggest a need for new structural models that can explain not only the credit spread puzzle but also the second moment variables. Another line of inquiry worth pursuing is to conduct a more rigorous and comprehensive analysis of finite-sample properties of the GMM test proposed in this study.

Appendix

Table AI. Summary statistics of individual names

This table reports credit ratings, 5-year CDS spread, equity volatility, leverage ratio, asset payout, and recovery rate, for each of the 93 firms similar to those by ratings and sectors in Table I.

Company	Last rating	Five year CDS (%)	Equity volatility (%)	Leverage ratio (%)	Asset payout (%)	Recovery rate (%)
Air Prods & Chems Inc.	A	0.238	28.358	33.067	2.086	40.863
Albertsons Inc.	BBB	0.692	35.540	54.662	3.650	41.008
Amerada Hess Corp.	BB	0.817	28.458	61.871	2.929	40.081
Anadarko Pete Corp.	BBB	0.427	31.244	47.816	1.688	39.439
Arrow Electrs Inc.	BBB	2.175	44.325	62.279	2.259	39.269
Autozone Inc.	BBB	0.708	33.269	30.222	0.827	41.977
Avon Prods Inc.	A	0.230	27.128	17.924	0.998	41.353
Baker Hughes Inc.	A	0.298	39.469	20.584	1.764	40.833
Baxter Intl Inc.	BBB	0.493	39.739	33.159	1.739	40.526
BellSouth Corp.	A	0.550	43.254	39.213	3.308	41.848
Black & Decker Corp.	BBB	0.389	29.569	45.897	1.566	42.200
Boeing Co.	A	0.517	36.815	56.877	1.744	39.336
BorgWarner Inc.	BBB	0.572	29.766	48.270	1.285	40.623
Bowater Inc.	BB	2.751	30.755	62.578	3.583	41.287
CSX Corp.	BBB	0.607	29.651	69.128	2.305	40.486
Campbell Soup Co.	A	0.319	27.171	36.114	2.699	40.063
Caterpillar Inc.	A	0.350	32.081	57.902	1.992	40.122

(continued)

Table AI. Continued

Company	Last rating	Five year CDS (%)	Equity volatility (%)	Leverage ratio (%)	Asset payout (%)	Recovery rate (%)
Cendant Corp.	BBB	1.595	42.626	59.864	1.291	39.440
Centex Corp.	BBB	0.895	41.148	69.613	2.543	40.670
Clear Channel Comms Inc.	BBB	1.413	45.192	35.378	1.487	40.789
Coca Cola Entpers Inc.	A	0.327	34.774	68.903	2.281	40.019
Computer Assoc Intl Inc.	BB	2.889	54.727	35.045	1.044	35.840
Computer Sciences Corp.	A	0.565	41.122	43.578	1.182	39.763
ConAgra Foods Inc.	BBB	0.470	27.510	43.829	3.516	39.320
Corning Inc.	BB	5.412	80.739	41.995	1.138	36.807
Delphi Corp.	BBB	1.470	40.828	77.164	1.535	40.539
Delta Air Lines Inc.	CCC	18.806	81.939	93.931	2.885	26.566
Devon Engy Corp.	BBB	0.732	31.487	56.495	2.281	40.513
Diamond Offshore Drilling Inc.	BBB	0.488	39.213	32.696	1.701	40.833
Dow Chem Co.	A	0.817	35.536	48.723	3.166	39.775
E I du Pont de Nemours & Co.	AA	0.241	30.318	37.916	2.574	41.409
Eastman Kodak Co.	BBB	1.317	37.618	56.431	2.550	38.839
Eaton Corp.	A	0.335	27.783	42.526	1.527	40.815
Electr Data Sys Corp.	BB	2.087	51.554	50.321	2.332	40.349
Eli Lilly & Co.	AA	0.219	35.486	13.956	1.898	40.494
Fedt Dept Stores Inc.	BBB	0.675	38.303	54.236	1.966	41.664
Ford Mtr Co.	BBB	2.977	47.060	92.612	2.769	41.849
GA Pac Corp.	BB	3.824	48.523	74.892	3.547	42.054
Gen Elec Co Inc.	AAA	0.427	36.356	63.713	2.223	40.883
Gen Mls Inc.	BBB	0.539	24.225	44.680	3.095	41.508
Gen Mtrs Corp.	BBB	2.434	35.537	94.017	2.595	41.278
Gillette Co.	AA	0.147	28.421	17.574	1.672	40.977
Goodrich Corp.	BBB	1.230	35.427	61.064	3.187	39.736
Goodyear Tire & Rubr Co.	B	7.671	65.509	88.106	2.245	39.840
H J Heinz Co.	A	0.310	23.404	39.061	3.199	41.748
Hilton Hotels Corp.	BBB	2.141	36.860	51.553	2.754	41.065
Home Depot Inc.	AA	0.222	39.170	14.502	0.741	42.223
IKON Office Solutions Inc.	BB	3.460	48.604	73.673	1.337	38.221
Intl Business Machs Corp.	A	0.381	31.166	32.683	0.578	39.991
Intl Paper Co.	BBB	0.740	30.566	58.274	2.944	39.674
J C Penney Co Inc.	BB	2.949	45.576	61.984	2.343	37.818
Jones Apparel Gp Inc.	BBB	0.634	32.547	26.906	1.353	41.338
Kerr Mcgee Corp.	BBB	0.745	26.472	59.613	3.398	41.242
Lockheed Martin Corp.	BBB	0.501	32.241	44.982	1.815	41.173
Lowes Cos Inc.	A	0.356	36.642	19.222	0.587	41.788
Ltd Brands Inc.	BBB	0.584	44.878	21.283	3.854	41.529
Lucent Tech Inc.	B	9.525	96.827	63.895	1.255	37.988
MGM MIRAGE	BB	2.167	33.197	57.910	2.675	39.764
Masco Corp.	BBB	0.612	33.101	35.400	2.758	42.234
Mattel Inc.	BBB	0.534	35.721	21.203	2.269	40.322
May Dept Stores Co.	BBB	0.608	36.953	52.074	3.923	41.765
Maytag Corp.	BBB	0.773	38.307	58.938	2.213	41.476
McDonalds Corp.	A	0.322	38.651	30.956	2.107	40.051

(continued)

Table AI. Continued

Company	Last rating	Five year CDS (%)	Equity volatility (%)	Leverage ratio (%)	Asset payout (%)	Recovery rate (%)
Nordstrom Inc.	BBB	0.609	40.304	43.145	1.555	41.820
Norfolk Sthn Corp.	BBB	0.471	36.021	61.054	2.704	39.724
Northrop Grumman Corp.	BBB	0.675	26.992	51.679	1.844	40.890
Omnicom Gp Inc.	BBB	0.906	36.220	42.475	0.887	40.262
PPG Inds Inc.	A	0.360	27.727	37.415	2.667	42.133
Phelps Dodge Corp.	BBB	1.780	38.034	48.840	1.877	41.547
Pitney Bowes Inc.	A	0.211	27.063	46.124	2.645	41.674
Praxair Inc.	A	0.291	28.048	33.167	1.730	42.060
Procter & Gamble Co.	AA	0.163	23.275	21.002	1.289	40.450
Rohm & Haas Co.	BBB	0.353	29.283	43.281	2.241	42.235
Ryder Sys Inc.	BBB	0.590	29.285	65.616	2.294	39.827
SBC Comms Inc.	A	0.598	43.723	42.509	3.587	38.423
Safeway Inc.	BBB	0.724	39.373	52.084	1.893	41.592
Sara Lee Corp.	A	0.281	28.465	42.474	2.900	39.904
Sealed Air Corp. US	BBB	2.349	35.792	44.043	1.820	37.390
Sherwin Williams Co.	A	0.396	29.004	32.345	1.896	41.694
Solectron Corp.	B	4.976	86.414	54.483	1.908	39.241
Southwest Airls Co.	A	0.723	43.900	29.447	0.624	40.323
The Gap Inc.	BB	2.889	50.769	27.086	1.429	41.034
The Kroger Co.	BBB	0.754	39.574	55.452	1.960	41.729
Tribune Co.	A	0.413	25.200	34.934	1.500	41.228
Utd Tech Corp.	A	0.260	30.856	37.047	1.116	39.475
V F Corp.	A	0.323	25.458	31.046	2.687	38.877
Valero Engy Corp.	BBB	1.075	36.741	65.574	2.174	40.715
Visteon Corp.	BB	2.671	46.160	87.957	1.297	41.348
Wal Mart Stores Inc.	AA	0.193	32.359	20.540	0.991	39.991
Walt Disney Co.	BBB	0.714	43.767	38.906	1.644	39.191
Weyerhaeuser Co.	BBB	0.753	29.759	62.255	3.509	41.164
Whirlpool Corp.	BBB	0.477	31.043	58.506	2.305	40.512
Williams Cos Inc.	B	6.836	84.181	83.953	3.724	35.851

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