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Price Change and Trading Volume: Behavioral Heterogeneity in Stock Market

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Abstract
The well-known Wall Street adage that states, "It takes volume to make prices move" has long suggested that there exists a positive correlation between absolute changes in stock price and trading volume. To practitioners who use technical analysis as their trading tool, trading volume has always been treated as a key signal to price change. Although many studies have empirically examined the nonlinear relationship between price change and trading volume, very few studies are able to provide a persuasive explanation for such price-volume relationship. This paper fills this gap by providing an explanation for such relationship under a framework of heterogeneous agent model with evolutionary switching mechanism. With the support of US stock market data, we first summarize some stylized facts on stock return and trading volume. We then mimic these facts using our model. The comparison between simulated and "real" time series shows that our model is not only able to replicate the seemingly chaotic fluctuations of the financial market but also able to explain how stock prices and trading volumes co-evolve with agents' belief.

Keywords Heterogeneous belief · Trading volume · Stock market · Stylized facts

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1 Introduction

In the analysis of financial market, much attention has been drawn to the study of assets price. Another important indicator of financial activities, trading volume, is ignored in many studies. Volume represents the total amount of transactions in a risky asset or entire market during a specific period of time. In the technical analysis, volume can be used to measure the relative worth of market move. A high volume during the price move always implies a strong market. The information roles of volume have been discovered by practitioners. As the old Wall Street adage asserts, “it takes the volume to move prices”.

The relationship between price and trading volume is an important topic in financial markets. However, few theories or models are developed to explain this relationship. As few paper discuss the characteristics of trading volume in the market, we try to generalize and summarize some stylized facts of trading volume first. Stylized facts like unit root and auto-regressive correlation can be easily tested using statistical tools, but several stylized facts can only be observed visually. These visual observed features are widely used in practice, so it is necessary to investigate them when study trading volume. One visualized stylized fact concerning volume is that a dramatic change in prices is always accompanied by significant volumes. This feature is frequently found in crisis periods. Figure 1 is the illustration of this phenomenon in 2008 financial crisis. We can clearly see that the Dow Jones Index experienced a sharp decline in the mid-2008 and fell to the bottom subsequently in the early 2009. At the same time the volume also surged and hit to an unprecedented level. Other similar examples are Black Monday in 1987 and Asian Financial crisis in 1997. The second visualized stylized fact is that trading volume is an important confirmation signal of price action in technical analysis. For example, if investors

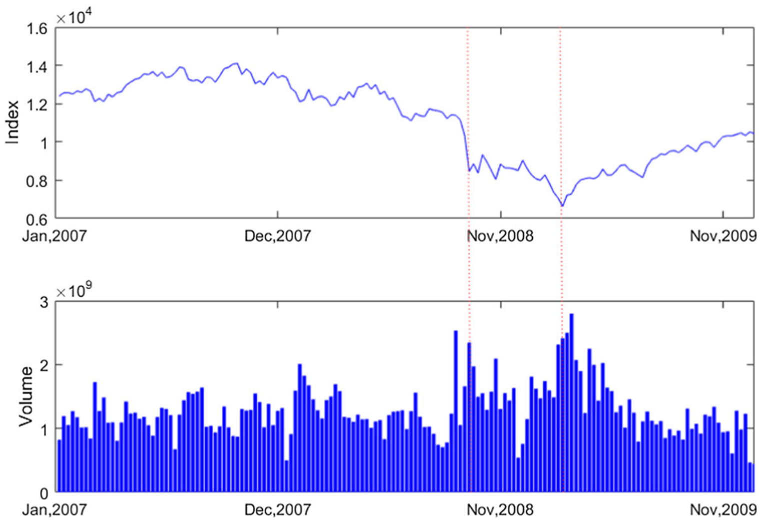

Fig. 1 Dow Jones index and trading volume during 2008 global financial crisis

observe a sudden increase in trading volume, they can confirm the breakout of price to the trend-line (see Murphy (1999) and Bulkowski (2011)). Figure 2 illustrates this fact in shares of Google. Its stock price fluctuated below the resistance line (around 575) for a long time until suddenly jumped up to around 660 in July 17, 2015. Meanwhile, the volume was almost five times the average value. After that the price moved up and down above the previous resistance level. The huge volume can be recognized as the signal to confirm that price breaks out the old regime and enters a new one.

In addition, the statistical relationship between price and volume has received considerable attention in a large amount of empirical research. Karpoff (1987) points out that volume could correlate positively with the elements of both the absolute change of price or the price change *per se*. Although an early empirical study by Granger and Morgenstern (1963) fails to find a correlation between price index and aggregated volume in New York Stock Exchange (NYSE) , succeeding studies have found evidence of positive correlation. Since 1990s, the dynamic (causal) correlation between price and volume has drawn a lot of attention. Bivariate vector autoregressive (VAR) models and Granger causality tests are used to investigate the price-volume relation. Saatcioglu and Starks (1998) find evidence that volume leads to price change. Statman et al. (2006) use monthly data from NYSE and find the positive relation between trading activities and lagged return. By using S&P 500 data from 1973 to 2008, Chen (2012) finds that trading volume does not Granger cause stock return, but return Granger causes volume. Apart from the linear model, Hiemstra and Jones (1994) apply nonlinear Granger causality tests to investigate the price-volume relationship in the US market. They find evidence of significant bidirectional nonlinear causality between returns and volumes. Moreover, Diks and Panchenko (2006) modify the method of nonlinear causality test to improve the performance, and find weak nonlinear causality between returns and volumes. Although much literature has proven that Granger causality exist between

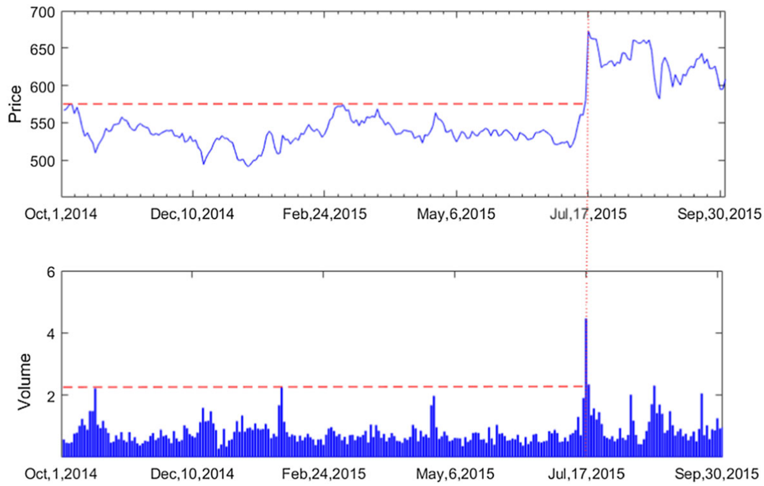

Fig. 2 Google stock and trading volume (2014–2015)

volumes and returns, the significance and directions are still controversial. To have a clearer view on price-volume relation, we reexamine the linear and nonlinear Granger causality between returns and volume changes. We use both S&P 500 and Dow Jones index daily data, the range of the sample is from 1/1/2010 to 12/31/2016. The method of linear and nonlinear Granger causality tests are provided in Appendix A and the results are reported in Tables 8 and 9, respectively. We find evidence that returns Granger cause volumes in the linear test, which is consistent with the results in Chen (2012). For the nonlinear test, we find weak Granger causality between specific lagged returns and volume changes in S&P500 data, but fail to find the causality relation in Dow Jones index. To further investigate the nonlinearities of stock price-trading volume relation, we refer to the information transfer in the literature. A recent study by Behrendt and Schmidt (2021) find the nonlinear information flow from returns to trading volume growth by using Shannon transfer entropy. The significant cross-correlation between stock price and trading volume is also found in Zhang and Shang (2021) in the framework of dispersion conditional mutual information (DCMI).

Beside the price-volume casual relation, it is also worth investigating the correlation between volume and volatility. Clark (1973) shows that the squared price changes are positively related to volumes. Daigler and Wiley (1999) find positive volume and volatility relation in the future market, and highlight that this relation is driven by the existence of different types of investors. In practice, the positive relation between volume and volatility can be easily found in current stock market. A recent example is the significant positive correlation between volatility and volume in US stock market during 2008 financial crisis. In practice, Chicago Board Options Exchange (CBOE) Volatility Index is frequently used to measure the stock market volatility. As shown in Fig. 3, the fluctuation of Volatility Index (VIX) is followed by the similar movement of S&P 500 trading volume. Just by eye-balling the trends of trading volumes and VIX in Fig. 3, there appears to be a strong inter-temporal correlation. To further check this correlation, we plot the correlation

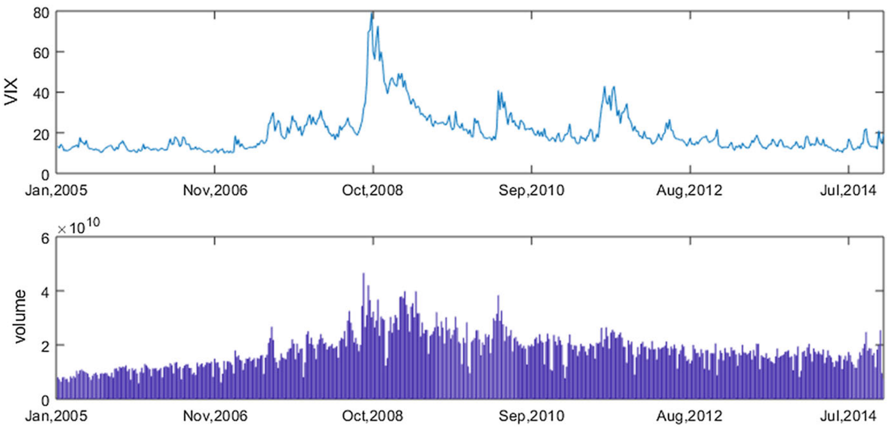

Fig. 3 S&P 500 volume and the VIX

chart and present it in Appendix C Fig. 21. We find a strong positive relation between volume and VIX with a significant coefficient correlation 0.725.

While ample studies focus on the detection of price-volume relationship, very few of them have explained why such patterns exist and how the relation evolves in the market. Even some theoretical studies attempt to explain this relationship, few models are able to address all the stylized facts as mentioned above. Due to the complexity of price-volume dynamic, nonlinear dynamic models have been drawn attention in the past decades. Granger (2014) argues that univariate and multivariate nonlinear models represent the proper way to model a real world that is almost certainly nonlinear. Since the sudden crisis in 1987, many studies have worked on the nonlinear models to detect and explain the well known stylized facts in financial markets. At the same time, behavioral finance with bounded rationality assumption rises to compete with traditional financial models with Efficient Market Hypothesis (EMH). As argued by Thaler (2005), behavior finance is a new approach to explain some abnormal regularities of financial markets, in response to the difficulties faced by traditional paradigm. The heterogeneous agent model (HAM) is one of the nonlinear behavioral models that has been widely used to explain complexity of financial markets, and it has been proven to be very powerful on asset pricing, replicating the stylized facts and explaining different features of financial market, such as crises, crushes and bubbles. Pioneering contributions in this direction include Beja and Goldman (1980), De Long et al. (1990), Day and Huang (1990), Brock and Hommes (1998), Lux (1995, 1998), Farmer and Joshi (2002), Chiarella and He (2003), Huang et al. (2010).

In this paper, we aim to build a HAM with trading volume. We mainly investigate how the prices, volumes and beliefs co-evolves in the stock market within fundamentalist-chartist framework. To evaluate the fitness of our model to real financial markets, we examine its capability of replicating the well known stylized facts in the market. As shown in the current literature, most of the HAMs have the ability to explain stylized facts concerning price or return. These stylized facts and strand of literature include: non-stationary price and stationary return (Hommes (2002)), fat tail (Lux (1998)),volatility clustering (Lux and Marchesi (2000), Hommes (2002)), leverage effect (Huang et al. (2013), Chen et al. (2013)), asymmetric returns (Huang et al. (2013)) and the power law of return (He and Li (2007), He and Zheng (2016)). Regrettably, little attention has been paid so far to volume except Brock and LeBaron (1995), Chen and Liao (2005), Westerhoff (2006) and Lespagnol and Rouchier (2018). Brock and LeBaron (1995) develops an adaptive beliefs model, and the model is able to reproduce the slowly decaying autocorrelation function of volatility and trading volume, but other stylized facts and co-evolvement of price and volume have not been fully investigated. Chen and Liao (2005) attempt to use an agent-based stock markets (ABMs) model to determine the price-volume series and reproduce the presence of the nonlinear Granger causality relation between the price and volume, but the simulation results are unpersuasive. Westerhoff (2006) builds a HAM by incorporating volume into chartist trading rule, but he just focuses on the price-volume signal for trading breaks. Lespagnol and Rouchier (2018) explore whether trading volume and price distortion can be explained by the investor's bounded rationality in the HAM framework.

To fill the gap of HAM on volume, we develop a simple HAM with trading volume. Comparing to previous HAMs, our model has greater potential to simulate the features of financial market. The contribution is threefold: First, our model is able to generate most of stylized facts both on price and volume. Second, the co-evolvement of prices, volumes and beliefs can be found in our simulation. Third, our model can explain the formation of different chart patterns, bubbles and crises, which provide theoretical support for technical analysis.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 presents the simulation results and verifies model's capability to explain market dynamics, including stylized facts, price dynamics, volume dynamics, bubble and crises formation and the co-evolvement of price changes, volumes and beliefs. Section 4 concludes.

2 Model

2.1 Heterogeneous Investors

Following the standard HAM literature (see for example Day and Huang (1990), Lux (1995), Brock and Hommes (1998), Chiarella and He (2003) Westerhoff and Dieci (2006) and Lux (2021)), we consider a market with only one risky asset and two types of agents, namely fundamentalists and chartists. We assume that these agents are bounded rational and hold heterogeneous beliefs on future price trend. While fundamentalists form their expectations on future price and adopt trading strategies based on market fundamental factors, such as dividends, profits and economic growth, chartists focus on technical analysis. They use historical price trends or chart patterns as essential elements to form their expectations and hence trading strategies. A market maker who adjusts market price in response to aggregate demand from fundamentalists and chartist is also included in the model.

2.1.1 Fundamentalists

Fundamentalists are assumed to have full information about fundamental value μ_t^f. They believe that asset price will not deviate from this value for long, and they expect price to fluctuate within a reasonable zone of (m_t, M_t) due to some short term external disturbances. The price will eventually converge to its fundamental value. As a result, fundamentalists buy assets when the price is below its fundamental value and sell otherwise. Accordingly, their demand function is written as:

$$D_t^f = \begin{cases} (\mu_t^f - p_{t-1})A(\mu_t^f, p_{t-1}), & p_t \in Q \\ 0, & p_t \notin Q \end{cases} \tag{1}$$

where $Q = [m_t, M_t]$ is the reasonable price zone, m_t and M_t are the minimum and maximum of the price boundaries, respectively and $A(\cdot)$ is a chance function with respect to μ_t^f and p_{t-1}. This function demonstrates the psychological behavior of investors Day and Huang (1990). It shows that when price is closer to the upper boundary of the price zone M_t, the chance of losing the existing gains increases. Likewise, when price is closer to the lower boundary of the price zone m_t, the chance of missing a capital gain by failing to buy is high[1]. Fundamentalists are assumed to update information Ω_t in each period to obtain more accurate fundamental value of the risky asset. Instead of using a constant fundamental value, we assume that they adjust the value according to the following rule:

$$\mu_{t+1}^f = g(t)\mu_t^f \tag{2}$$

where $g(t)$ is the simulated business cycle such that:

$$g(t) = \begin{cases} g & t \in [4(i-1)\cdot s, (4i-1)s] \\ -\frac{g}{2} & t \in [(4i-1)s, 4i \cdot s] \end{cases} \qquad i = 1, 2, 3 \cdots \tag{3}$$

where g is the economic growth rate. We assume that a business cycle consists of $4s$ periods. During economic boom, the economy expands at an average growth rate of g percent for $3s$ periods. Then a recession kicks in and lasts for $1s$ period with a growth rate of $-g/2$. This kind of seasonal cycle has been discussed in Shapiro and Watson (1988). Although it is not a rigorous assumption, with this setting up, macroeconomics factors are able to be incorporated into the model and hence improves the ability of the model in explain the relationship between business cycle and stock market cycle.

2.1.2 Chartists

Unlike fundamentalists who make their trading decisions based on fundamental values, chartists trade mostly based on the past price trend of the risky asset. As a result, they estimate the short-term fundamental value v_t based on historical price and then extrapolate the next period market price by the deviation of price from its short-term fundamental value. This deviation is also known as estimation bias. Accordingly, chartists form their expectation based on the following rule:

$$E_{t-1}^c(p_t) = p_{t-1} + \beta(p_{t-1} - v_{t-1}) \tag{4}$$

where $\beta \neq 0$ measures the sensitivity of price expectation to the latest bias which is denoted by $(p_{t-1} - v_{t-1})$. Thus, the demand function of chartists is given by:

$$D_t^c = \eta[E_{t-1}^c(p_t) - p_{t-1}] = \eta\beta(p_{t-1} - v_{t-1}) \tag{5}$$

where η denotes the elasticity of the demand function of chartists. For simplicity, we further assume that all chartists share the same belief on the short-term fundamental value v_t. However, due to opinion differences, chartists may choose to follow or against the price trend. Such differences between chartists are captured by the sign of β. For those who believe that the market price p_t will deviate further away from

[1] More details about the chance function can be found in Appendix B.

p_{t-1}, $\beta = \beta_1 > 0$ and the price trend will persist. They are said to be trend followers who hold on to positive bias. Accordingly, their excess demand can be expressed as:

$$D_{1,t}^{c} = \eta_1 \beta_1 (p_{t-1} - v_{t-1}) \tag{6}$$

This equation means that when $p_{t-1} > v_{t-1}$, trend followers with $\beta_1 > 0$ would believe that the price trend will continue and hence buy in ($D_{1,t}^{c} > 0$).

On the contrary, for chartists who believe that the price trend will reverse in the next period, then $\beta = \beta_2 < 0$. These chartists are known as contrarians as they take negative bias in making their trading decision. As a result, their excess demand function is taken as:

$$D_{2,t}^{c} = \eta_2 \beta_2 (p_{t-1} - v_{t-1}) \tag{7}$$

Thus, when $p_{t-1} > v_{t-1}$, contrarians with $\beta_2 < 0$ would think that the price trend will reverse and choose to sell ($D_{2,t}^{c} < 0$).

We now look more closely at the short-term assets value, v_t. We assume that all chartists, both trend followers and contrarians, hold on to an identical short-term assets value. As in Huang et al. (2010) and Huang and Zheng (2012), we assume that chartists adopt the adaptive belief mechanism where they update their expectation on short-term assets value. They believe in support and resistance levels which are derived from common rules of technical analysis[2]. We assume that chartists divide price domain $P = [p_{min}, p_{max}]$ into n regimes such that:

$$P = \bigcup_{j=1}^{n} P_j = [\bar{p}_0, \bar{p}_1) \cup [\bar{p}_1, \bar{p}_2) \cup \cdots \cup [\bar{p}_{n-1}, \bar{p}_n] \tag{8}$$

where $\bar{p}_j$ for $j = 1, 2, \cdots, n$ represents the different support and resistance levels chartists set.

The short-term fundamental asset value can be simply extrapolated as the average of the top and the bottom threshold prices:

$$v_t = (\bar{p}_{j-1} + \bar{p}_j)/2 \quad \text{if} \;\; p_t \in [\bar{p}_{j-1}, \bar{p}_j) \text{ and } j = 1, 2 \cdots n \tag{9}$$

When price fluctuates within the current regime, there are enough reasons for chartists to believe that the short-term fundamental asset value will remain unchanged. However, once price breaks through either the support line or the resistance line, chartists will adjust their expectation on the short-term fundamental asset value according to Eq. (9). This regime switching phenomenon is commonly found in stock market with chartist's beliefs evolve with regime switching. According to Huang et al. (2010), the short-term fundamental value for each period is estimated as:

$$v_t = (\lfloor p_t/\lambda \rfloor + \lceil p_t/\lambda \rceil) \cdot \frac{\lambda}{2} \quad \text{if} \;\; p_t \in [\bar{p}_{j-1}, \bar{p}_j) \text{ and } j = 1, 2 \cdots n \tag{10}$$

[2] Donaldson and Kim (1993) have provided empirical evidence of the existence of support and resistance levels in Dow Jones Industrial Average.

2.2 Evolutionary Belief Switch

In our model, we assume that agents can switch their strategy and the switching between strategies is driven by discounted expected profit which is denoted by π. In each period, market then updates according to the switching between strategies by different types of agents. Specifically, the discounted expected profit functions of fundamentalist and chartists (trend followers and contrarians) are written as:

$$\begin{aligned} \pi_{1,t}^c &= |\beta_1(p_{t-1} - v_{t-1}) - rp_{t-1}| \\ \pi_{2,t}^c &= |\beta_2(p_{t-1} - v_{t-1}) - rp_{t-1}| \\ \pi_t^f &= s(p_{t-1})|(u_t^f - (1+r)p_{t-1})| - C \end{aligned} \tag{11}$$

where $\pi_{1,t}^c$, $\pi_{2,t}^c$ and π_t^f are expected profit of trend followers, contrarians and fundamentalists, respectively. r is the interest rate between period $t-1$ and period t. C is the information cost which fundamentalists have to incur to acquire additional information. $s(p_t)$ is the discount factor. For chartists, as they are only interested in one-period returns and capitalize gains or losses immediately, we assume that they have a discount factor of 1. While fundamentalists value assets according to the cash flows that the asset is expected to generate. These agents can be thought of as following a buy-and-hold strategy and they take more time to capitalize their gains or losses, so the discount factor for them is assumed in the form of $s(p_t) = |(u_{f,t} - p_t)/3u_{f,t}|$.

We let $\omega_{i,t}$ denote market fraction of the three different types of investors. The fractions of the three groups vary endogenously over time according to choice model with multinomial logit probabilities introduced by Brock and Hommes (1998):

$$\omega_{i,t}(p_t) = \frac{exp(\rho\pi_{i,t}(p_t))}{\sum_k exp(\rho\pi_{k,t}(p_t))} \tag{12}$$

where the parameter ρ is the intensity of choice which measures the speed of transition between different beliefs. A high value of ρ means that more investors will switch between strategies. $\omega_{i,t}$ is always positive which implies that not all agents are going for strategy with high discounted expected profit. The belief updating or switching behavior has been widely discussed in behavioral finance literature, such as Hirshleifer (2001), Ko and Huang (2012) and Anufriev et al. (2013).

2.3 Price Determination

Another participant is market marker who mediates transactions in the market to provide liquidity. The market marker collects orders from all traders, namely trend followers, contrarians and fundamentalists, then sets the price. The market maker supplies from his inventory when there is a positive excess demand and do the reverse otherwise. In each period, market price[3] is updated adaptively according to the following adjustment rule:

$$p_t = p_{t-1} + \gamma(\omega_{1,t}D^c_{1,t} + \omega_{2,t}D^c_{2,t} + \omega_{3,t}D^f_t) \tag{13}$$

where γ is the speed of price adjustment to excess demand.

2.4 Volume Formation

Besides price, volume is another important indicator in stock market. The total number of all shares transacted in the market is known as trading volume. In our market maker framework, we assume that chartists with different expectation on future price trade with each other first, then together they generate either a positive or a negative excess demand. Compared to chartists, fundamentalists hold totally different strategy and they may have either a positive or a negative excess demand. Putting them together, we can foresee two scenarios. First, if fundamentalists and chartists share the same opinion on the future price trend, then both groups should trade directly with market marker. Trading volume is the absolute value of aggregate demand. Second, if fundamentalists and chartists hold different opinion on the future price trend, they will trade with each other first. Market maker will only come in to satisfy the remaining excess demand. In this case, trading volume is the maximum of the absolute value of the excess demand for each group. As a result, trading volume is defined as:

$$V(p) = \begin{cases} min(|\tilde{D^c}_{1,t}|, |\tilde{D^c}_{2,t}|) + |\tilde{D^f}_t + \tilde{D^c}_{1,t} + \tilde{D^c}_{2,t}| & \text{if } \tilde{D}_{f,t}(\tilde{D^c}_{1,t} + \tilde{D^c}_{2,t}) > 0 \\ min(|\tilde{D^c}_{1,t}|, |\tilde{D^c}_{2,t}|) + max(|\tilde{D^f}_t|, |\tilde{D^c}_{1,t} + \tilde{D^c}_{2,t}|) & \text{if } \tilde{D}_{f,t}(\tilde{D^c}_{1,t} + \tilde{D^c}_{2,t}) < 0 \end{cases} \tag{14}$$

where $\tilde{D^c}_{1,t}$, $\tilde{D^c}_{2,t}$ and $\tilde{D^f}_t$ are the weighted excess demand for trend followers, contrarians and fundamentalists, respectively. $\tilde{D^c}_{1,t} = \omega_{1,t}D^c_{1,t}$, $\tilde{D^c}_{2,t} = \omega_{2,t}D^c_{2,t}$ and $\tilde{D^f}_t = \omega_{3,t}D^f_t$.

3 Simulation Results

In this section, we discuss simulation results from the HAM framework with an additional feature of trading volume that we have elaborated earlier. By putting together prices, trading volumes and agents' beliefs, we are able to generate richer simulation results from the model. We first evaluate the fitness of our model in capturing the complex dynamics of stock market by exploring the power of the model in reproducing some of the well-documented stylized facts that were mentioned in Sect. 1, keeping in mind that no single model can, or should, fit most aspects of the data but recognize instead that some consistency can still be useful. Besides the price related stylized facts, we also investigate whether the simulated

[3] Logarithmic price could also be used to scale the price.

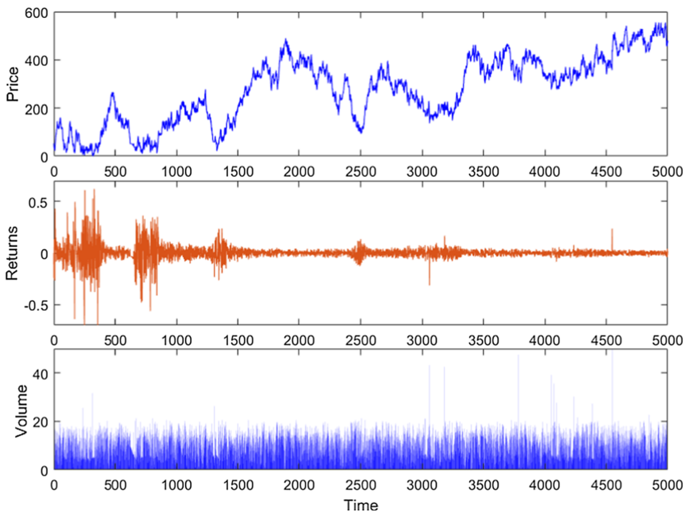

Fig. 4 Simulated time series of price (top), returns (middle) and volume (bottom)

dynamics of the model are consistent with the stylized facts on trading volume. Specifically we look at (1) the correlation between volume and volatility, (2) Granger causality between volume and returns, (3) information role of volume and (4) volume and chart patterns. We then examine both price and trading volume dynamics and see how these variables co-evolve with agents' beliefs under different chart patterns and financial crisis.

In order to maintain consistency and unity, we simulate our price and volume series using a uniform set of parameters throughout the paper unless otherwise specified. Specifically, we set $\mu_1^f = 50$, $d_1 = d_2 = -0.3$, $k = 2$, $s = 25$, $\lambda = 13.1787$, $C = 3$, $r = 10^{-4}$, $g = 0.008$, $a = 1$, $\beta_1 = 1.2$, $\beta_2 = -0.7$, $\eta_1 = 0.833$, $\eta_2 = 3.214$, $\rho = 0.9$ and $\gamma = 1$. To see how fit the simulated data is in matching the actual financial time series in terms of its statistical and qualitative properties, we take the daily price and trading volume of S&P 500 and Dow Jones Index as benchmark. The time range of these dataset is from 1 January 2010 to 31 December 2016. Stock returns are expressed in percentage and hence taken as $log(p_t/p_{t-1}) \times 100$ and percentage change in trading volume is taken as $log(v_t/v_{t-1}) \times 100$. As some stylized facts may be sensitive to the selection of sample period, we also refer to other literature in finance to verify the explanatory power of our model.

Before we explore the simulated stylized facts, we first look at the simulated price, returns and volume dynamics of our model in Figure 4. Generally we see that stock price follows a random walk with returns fluctuate around zero and tend to cluster. The largest positive and negative returns demonstrate different magnitudes, implying that there exists asymmetry in stock returns. Trading volume changes with price and returns and demonstrates occasional spikes which suggest abnormal large trading volume at times.

The top panel of Fig. 5 displays the simulated prices and market fundamental value which is computed based on Eq.(3). We classify periods with market prices

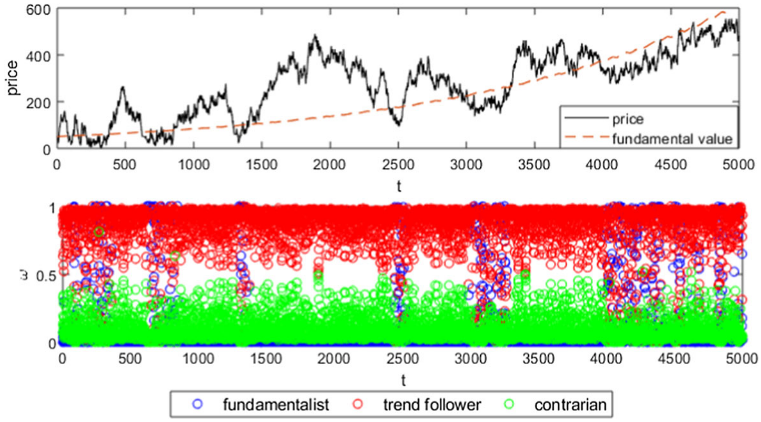

Fig. 5 Simulated prices and fundamental values (top) and agents' fractions (bottom)

above (below) its fundamental values as bubbles (busts). The remaining panels show the corresponding fractions of agents in the market with ω_1, ω_2 and ω_3 indicate fraction of trend followers, contrarians and fundamentalists in the market, respectively. The value of fraction ranges between 0 and 1. When $\omega_{i,t}$ approaches 1, agents i are said to dominate the market at time t. When market prices deviate away from its fundamental values, we see some endogenous self correction where agents switch between different types of trading strategies.

3.1 Price Dynamic

Our model is a nonlinear model. The nonlinearity in the model is mainly due to the setup of the chance function $A(\mu_t^f, p_{t-1})$ and the way chartists compute the market's short-term fundamental values. The chaotic system developed has the capability of generating relatively rich price dynamic patterns which are commonly observed in stock market. For more insights, we show a typical phase diagram of the simulated prices in Fig. 6. The 45 degree line shows that $p_t = p_{t-1}$. From the figure, chaotic multi-phase switching of prices is observed. Prices fluctuate across the 45 degree line several times suggest that there exists multiple equilibria. As introduced by Huang and Zheng (2012), these price patterns can be classified into a rising zone and a declining zone. When prices are above the 45 degree line, it would rise in the next period. Likewise, if the prices are below the 45 degree line, it is expected to fall. By combining the market conditions and investigating further into two step-wise dynamics, Huang and Zheng (2012) further classify the declining zones into sudden, smooth and disturbing declining zones, and discuss the mechanisms and conditions of how prices continue to stay within the same regime or escape from one regime to another.

As bifurcation diagrams are frequently used to illustrate the dynamic properties of a nonlinear system, we also run a number of numerical simulations to illustrate how the local dynamics of our model vary with some important parameters, such as

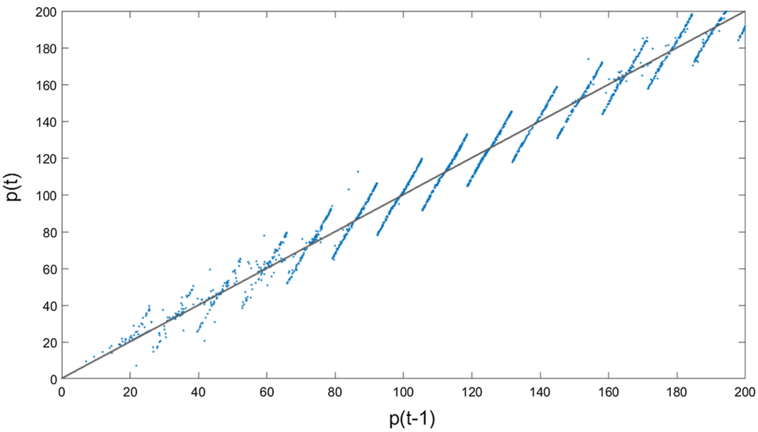

Fig. 6 The phase diagram of price

the speed of price adjustment (γ), the intensity of choices (ρ) and other exogenous parameters which include the sensitivity of fundamentalists when price moves close to the boundaries of m_t and M_t (a), the sensitivity of the price expectation of trend followers to the bias (β_1), the sensitivity of the price expectation of contrarians to the bias (β_2), and information cost of fundamentalists (C). Unless otherwise stated, in all bifurcation diagrams, bifurcation parameter is increased in 2000 discrete steps while keeping all other parameters constant.

As discussed earlier, our model has multiple equilibria due to multiple-regimes setup for chartists. Unlike Huang and Zheng (2012) who look at regime-switching dynamics, our discussion focuses on within-regime dynamics. In doing so, we set the short-term fundamental value v_t to 50 and growth rate to 0 in our simulations. In our first analysis, we examine the effect of the speed of price adjustment (γ) on the price dynamics when the intensity of choice, $\rho = 0.85$ as in panel (a) and $\rho = 1.85$ as in panel (b), respectively. In panel (a) of Fig. 7, we set ρ to 0.85 and vary γ from 0 to 3. We can see that the bifurcation evolves from a stable steady state to a chaotic price fluctuation when γ increases from 0 to 3. The primary bifurcation toward instability is a period-doubling bifurcation at which the steady state becomes unstable and a stable 2-cycle emerges. This happens when the value of γ is approaching 1.7. When γ continues to increase, further bifurcations occur and the price dynamics become increasingly more complicated. Likewise, in panel (b) of Fig. 7, we use ρ of 1.85 and vary γ from 0 to 3 to explore the effect of the intensity of choice on the price dynamics. We see that with a larger ρ, the primary bifurcation occurs at a much lower value of γ of around 1. Besides, price fluctuations are more complicated with higher price adjustment speed, i.e. larger value of γ.

Panels (c) and (d) are results from our second analysis where we study the effect of the intensity of choice (ρ) on the price dynamics when the speed of price adjustment is set at 1 and 2, respectively. The bifurcation in panel (c) is very similar to that of panel (a) which suggests that an increase in the intensity of choice will trigger a rational route to randomness. In panel (d), we repeat these simulations for

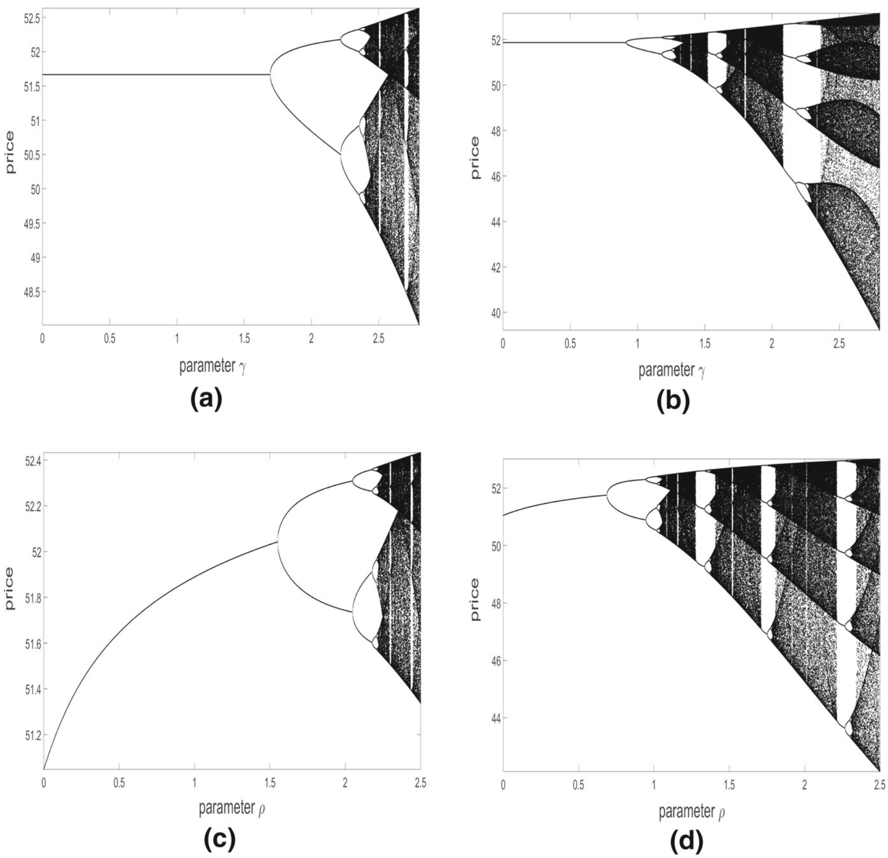

Fig. 7 Dynamics of the model. **a** Bifurcation diagram for speed of price adjustment with $\rho = 0.85$. **b** Bifurcation diagram for speed of price adjustment with $\rho = 1.85$. **c** Bifurcation diagram for intensity of choice with $\gamma = 1$. **d** Bifurcation diagram for intensity of choice with $\gamma = 2$

$\gamma = 2$ and find that the primary bifurcation toward instability occurs at low intensity of choice of 0.7 in panel (d) as compared to 1.6 in panel (c). The price dynamic shows higher amplitude fluctuation with higher intensity of choice.

More bifurcation diagrams that show how the asymptotic dynamics vary with other exogenous parameters including the sensitivity of fundamentalists when price moves close to the boundaries of m_t and M_t, the sensitivity of the price expectation of trend followers to the bias, the sensitivity of the price expectation of contrarians to the bias, and information cost of fundamentalists are drawn in Appendix D.

3.2 Stylized Facts on Price

3.2.1 Unit Root

As been proven in the finance literature, the series of stock prices are not stationary but returns and volumes are usually stationary. To examine whether our model is compatible with this stylized fact, we check the unit root of SP&500 Dow Jones Index data as well as our simulated dataset, and the results are displayed in the Table 1. The augmented Dickey-Fuller(ADF) are used to test the existence of unit root. The ADF for three price time series are − 0.661, − 0.675 and − 2.158, which are significantly greater than the critical value at 10% significant level. The corresponding p-values imply that all three price statistic are unit root process. In the return and volume series, the ADF tests significantly reject the null hypothesis of non-stationary at 1% level for all series. Therefore, we are confident that our simulation match the actual financial data well in the terms of stationary for price, return and volume.

3.2.2 Fat Tails

Fat-tailed distributions of financial asset returns are well documented in empirical studies (see Cont (2001), Chakraborti et al. (2011) and Eom et al. (2019)). The fat tail of return suggests that extreme returns appear more frequently than what are predicted by the normal distribution. To investigate the fat tails of the simulated return, we calculate the skewness and kurtosis values and compare them with that of actual financial data. Table 2 summarizes the statistic of fat tail tests on our simulation, S&P 500 and Dow Jones Index. The kurtosis (the fourth moments) for our simulation is 10.514, which is close to the value for actual financial time series. The kurtosis for all three series are positive and greater than the benchmark value 3 when returns are normally distributed. It is consistent with finding in literature that distribution of returns displays a fat tail with positive excess kurtosis. The skewness (the third moments) of simulated returns is -0.440, which is very close to the skewness of the well-known indexes. The negative value of skewness implies that the price falls more than it rises on average.

Table 1 Unit root test

Statistic	Price		Return		Volume	
	ADF	*p*-value	ADF	*p*-value	ADF	*p*-value
S&P 500	− 0.661	0.857	− 44.010	0.000	− 18.742	0.000
Dow Jones Index	− 0.675	0.853	− 44.037	0.000	− 21.444	0.000
Our simulation	− 2.158	0.222	− 37.604	0.000	− 31.645	0.000

Notes: ADF critical value are:− 3.43(1%), − 2.86(5%), − 2.57(10%)

Table 2 Fat tails

Statistic	Skewness	Kurtosis
S&P 500	− 0.424	7.202
Dow Jones Index	− 0.385	6.526
Our simulation	− 0.440	10.514

3.2.3 Volatility Clustering

Another robust statistical property of financial market is the existence of volatility clustering. The volatility clustering phenomenon refers to significant changes of prices tend to cluster together, resulting in persistence of the amplitudes of price changes. Cont (2007) reveals that the different behavior of agents is the reason causes the volatility clustering. In order to check whether our model is able to account for this stylized fact, we plot the autocorrelation functions (ACF) for both artificial data and S&P 500 data.

The returns show almost insignificant autocorrelations for both actual data and artificial data in Fig. 8. No linear correlation does not mean independence of return. A different picture emerges when one takes non-linear functions of return into account, such as absolute or squared returns. As shown in Fig. 8, the ACFs of simulated absolute returns and squared returns are slowly decaying as time lag increases. The persistent positive correlation is a quantitative feature of volatility clustering, meanwhile it also implies a long range dependence that one typically finds in financial time series. The slowly decaying patterns in the ACFs of artificial

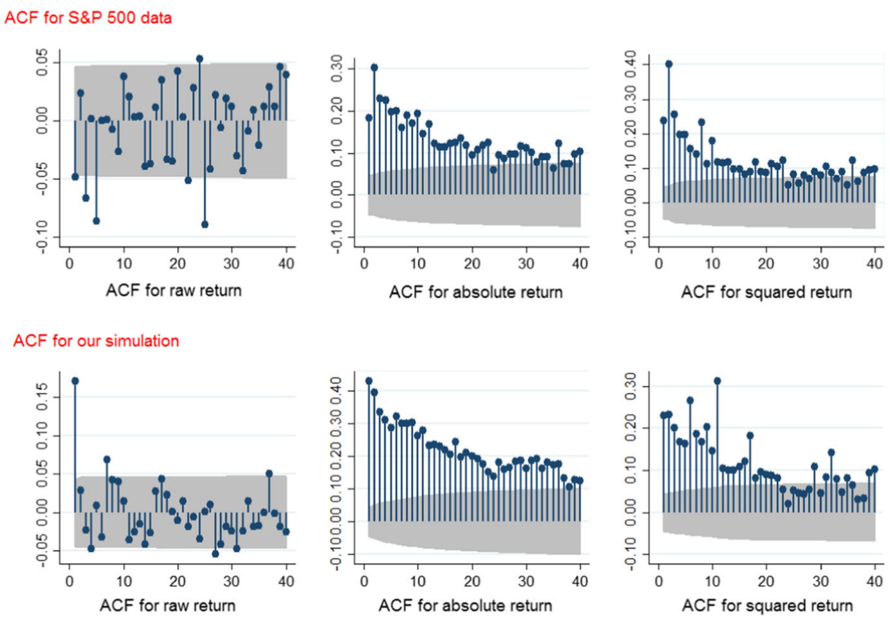

Fig. 8 Autocorrelation functions for raw return, absolute return and squared return

data are analogous to that for S&P 500 data, which demonstrates that our model performs quite well in generating the important stylized facts in financial market.

3.2.4 Asymmetric Returns

Asymmetric returns is another stylized fact in financial market. In our simulated sample, the most positive return is 44% while the most negative return is – 61%, which matches the documented asymmetry in returns. To test the asymmetry of the statistic, we run the Shapiro-Wilk test for normality. The null hypothesis for this test is that return is normally distributed. The result in the Table 3 significantly rejects the null hypothesis, and the distribution of simulated returns does not follow a normal distribution. We also do the symmetric plot for the simulated return, and some points departing from the 45 degree line in Fig. 9 strongly implies that the returns are asymmetric.

3.2.5 The Power Law of Returns

The tail distribution of returns can be well approximated by the power law, which has been found and investigated in many literature (Gabaix et al. (2003), He and Li (2007), Lux and Alfarano (2016), Schmitt and Westerhoff (2017)). In particular, the distribution of returns is found to decay according to

$$P(|r_t| > x) \sim X^{-\alpha} \tag{15}$$

where α is a constant parameter of the distribution known as scaling parameter or tail index. To detect the power law distribution of return, we follow the method as in Clauset et al. (2009), and the method involves maximum-likelihood fitting methods with goodness-of-fit tests based on the Kolmogorov-Smirnov statistic and likelihood ratios. As the power law distribution of return is sensitive to the frequency of sample

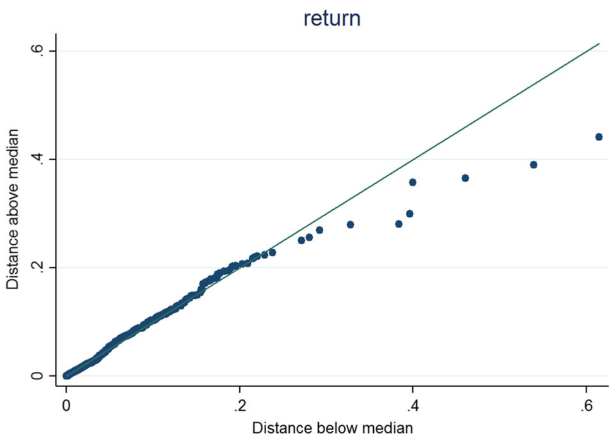

Fig. 9 Symmetric plot of return

and time range, we test the daily data (2010/1/1-2016/12/31) and weekly data (1983w1-2016w52) for both S&P 500 and Dow Jones Index, and find that the weekly data of these two indexes follow the power law distribution. To investigate whether our model could generate data with the power law distribution, we test the simulated data and find that p-value equals to 0.13, which suggests one can not reject the null hypothesis that data is generated from a power law distribution. The details of the results are shown in Table 4.

3.3 Volume Dynamics

Before we turn to the stylized facts on volume, we first explore the statistical properties of our simulated volumes. Figure 10 turns to the statistics of simulated volumes. The top panel of the diagram characterizes the distribution of simulated volumes and normally distributed volumes. As can be seen, the distributions of simulated volumes deviate significantly from normal distribution and possess fat tails. The bottom panel of Fig. 10 exemplifies the autocorrelation function of simulated volumes,and the persistence of ACF goes out for a long range and the rate of decaying is very low, which quantifies the long memory behavior of trading volumes.

Another statistical feature in financial market is convex or V-shaped relationship between price and trading volume. As reported by Karpoff (1987), a V-shape has been found by virtually all empirical investigation of the price-volume relation in equity market. This V-shape curve has also been widely documented in price-volume relation studies, such as Gallant et al. (1992), Blume et al. (1994) and Puri and Philippatos (2008). By building a model for price and trading volume in financial market, Gallant et al. (1992) found that the dispersion of the information affects this V-shape relation, but in our model the effect of the disagreement of investor's beliefs is perhaps even more interesting. We plot the price-volume outcomes for simulated data from our model, and we explored the possible effect of disagreement of belief from two chartists groups (sensitiveness disagreement is measured by $\beta_1 - \beta_2$). Figure 11 depicts the resulting price-volume equilibrium

Table 3 Shapiro-Wilk test

Variable	Observations	W	V	z	Prob> z
Return	2000	0.929	83.850	11.266	0.000

Table 4 The power law of return

Statistic	X_{min}	α	p-value
S&P 500(weekly)	3.45	3.83	0.77
Dow Jones Index(weekly)	4.07	4.44	0.82
Our simulation	0.10	3.90	0.13
Test hypothesis:			
H_0 : data is generated from a power law distribution.			
H_1 : data is not generated from a power law distribution.			

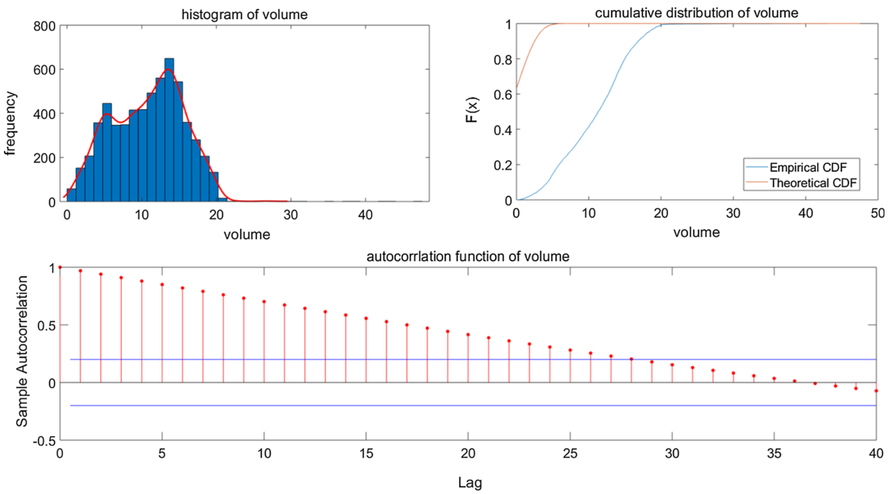

Fig. 10 Statistics of simulated volume

outcomes for three different level of disagreements of chartists. What is most interesting is that while the greater disagreement of chartists increases the disagreement of the points (as from panel (a) to (c)), it does not change the general V-shape of the relation. Indeed, the graph suggests when the disagreement of the chartists' belief is small enough, the price-volume relation is characterized by a simple V-shape. As volume also contains information of investor's beliefs and volume statistic provides information to the market that is not conveyed by price, observing price and volume together is more informative than observing price alone.

3.4 Stylized Facts on Volume

As mentioned before, most of the HAMs have the ability to generate the stylized facts on price, but few have investigated the performance of HAM on generating volume related stylized facts. In this section, we will test our model about the capability of replicating some stylized facts on trading volume, which have been mentioned in Sect. 1. While some of them can be detected by statistic approaches, others will rely on the visualized analysis.

3.4.1 Correlation between Volume and Volatility

As shown in Section 1, there exists a strong positive linear relation between volume and VIX. Using the same method, we test the correlation between the simulated trading volume and volatility. Squared returns serve as the proxy of volatility. The correlation coefficient for the test is 0.337 (Fig. 12), which suggests a weak positive correlation. Although the correlation between simulated series is not as strong as that in actual dataset, the significant positive correlation is consistent with the

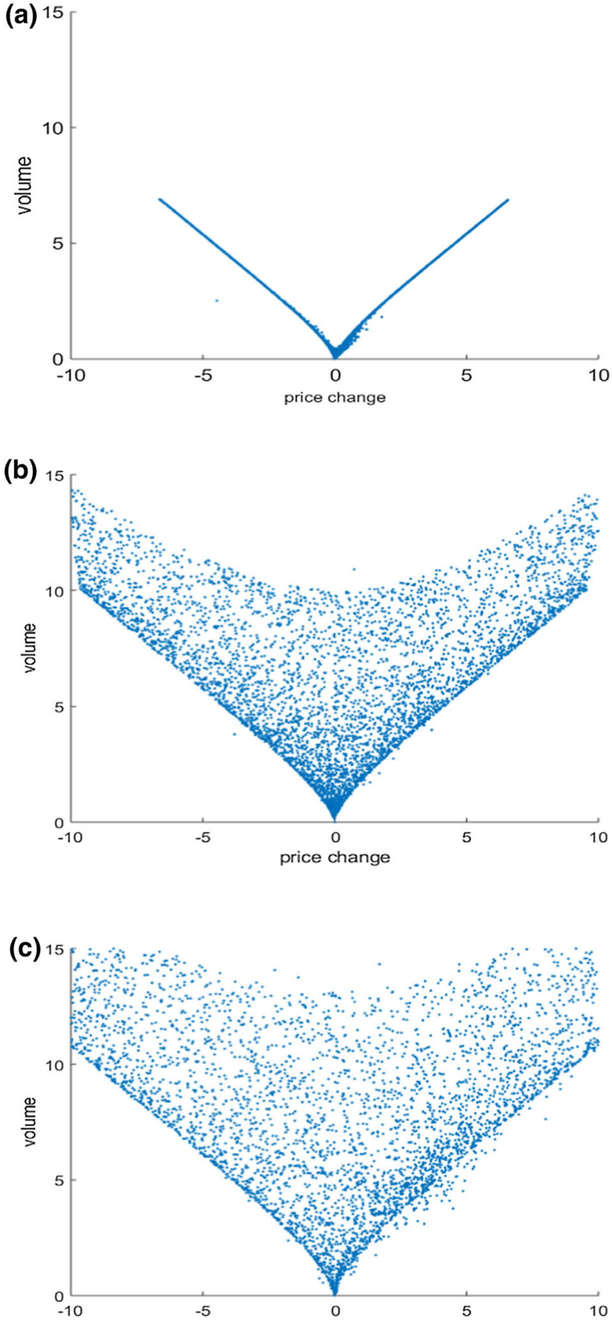

Fig. 11 The relation of price and volume for different disagreements of chartists belief. **a** Disagreements of chartists belief $\beta_1 - \beta_2 = 2$. **b** Disagreements of chartists belief $\beta_1 - \beta_2 = 3$. **c** Disagreements of chartists belief $\beta_1 - \beta_2 = 4$

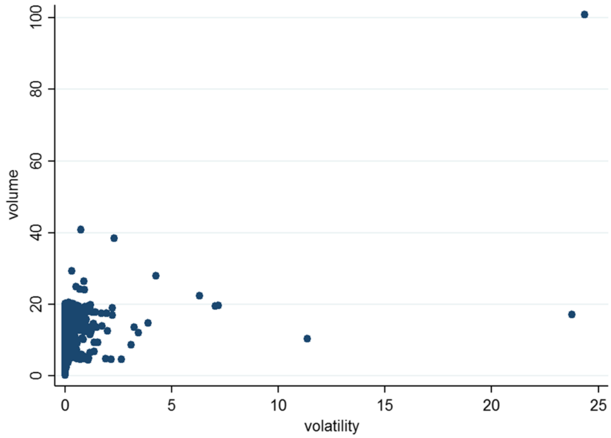

Fig. 12 Correlation between volume and volatility

literature. In the past decades, the mechanism of positive correlation between volume and volatility has been always discussed by many researchers, the heterogeneous beliefs of agents in our model could provide a reasonable explanation for this question. As stated in Shalen (1993), the dispersion of beliefs is a factor contributing to the positive correlation between volume and volatility.

3.4.2 Granger Causality between Return and Volume Change

The linear and nonlinear relations between price and volume have been found in different markets and countries. To further examine the validity of our model, we conduct the linear and nonlinear Granger causality tests to investigate whether the causality relations exist in our simulation. The methods we use to test the linear and nonlinear Granger causality are the same with those in Sect. 1. The results are reported in Tables 5 and 6, respectively. In the linear Granger causality test, four lags for dependent and independent variables are considered in the VAR model. The Granger tests show strong evidence of unidirectional causality from return to volume changes. In particular, the Wald statistic value equals 17.829, and suggests one can confidently reject the null hypothesis of no causality at 1% significant level. On the other hand, Granger noncausality from volume change to stock return can not be rejected, as the p-value of the test 0.919 is great enough. Hence, unidirectional causality from stock return to trading volume exists in our simulation.

To check whether the nonlinear relation exists in the artificial series, we further conduct the nonlinear Granger causality tests for return and volume change. We follow the setup for lags and other parameters in Sect. 1, and the results in Table 6 show that all the t-statistics are larger than 1.645 (5% significant level), which implies there are strong evidence that one can reject the null hypothesis. Bidirectional nonlinear Granger causality between stock returns and volume changes is found in our simulation. The linear and nonlinear test results we get from the simulation are roughly consistent with the results using above S&P 500 and

Table 5 Linear Granger causality test

Dependent varibale	(1) R_t	(2) V_t
Constant	0.005	1.438***
	(0.009)	(0.074)
R_{t-1}	0.237***	0.183
	(0.022)	(0.167)
R_{t-2}	0.065	0.565***
	(0.023)	(0.172)
R_{t-3}	− 0.002	− 0.182
	(0.023)	0.172
R_{t-4}	− 0.145***	0.076
	(0.022)	(0.168)
V_{t-1}	− 0.002	0.337***
	(0.003)	(0.022)
V_{t-2}	− 0.001	0.045
	(0.003)	(0.024)
V_{t-3}	0.002	− 0.050***
	(0.003)	(0.024)
V_{t-4}	− 0.001	0.021
	(0.003)	(0.022)
Wald-stat	0.937	17.829***
p-value	0.919	0.005

Notes: (1) The entries in brackets are the standard errors. The Wald-stat and p-value are tests of Granger causality. (2)* denotes rejection at the 10% level, ** rejection at the 5% level, ***rejection at the 1% level

Table 6 Nonlinear Granger causality test

Lags	H_0:Stock Returns Do not Cause Volume Changes		H_0:Volume Changes Do not Cause Stock Returns	
Lx=Ly	T-statistic	p-value	T-statistics	p-value
1	1.654	0.049	3.393	0.000
2	1.733	0.042	2.990	0.001
3	1.976	0.024	3.303	0.000
4	2.180	0.015	2.211	0.014
5	2.190	0.014	2.348	0.009
6	1.806	0.035	2.177	0.015
7	1.895	0.029	1.678	0.047
8	1.818	0.034	1.811	0.035

*Note*Test critical value for T statistics are 2.326 (1%),1.645 (5%) and 1.282 (10%)

Dow Jones index dataset. Referring to the literature, the results are exactly consistent with the findings in Hiemstra and Jones (1994) both in linear and nonlinear Granger causality tests. The results also justify that our model is capable of simulating the relationships between returns and volume changes in stock market.

3.4.3 Significant Volume with Large Price Change

There are some visualized stylized facts on trading volume which have drawn much attention from practitioners and researchers. The first one we intend to simulate is that significant volume is always associated with price jump. Besides the statistical relationship we have tested using econometric approaches, the visualized relationship between prices fluctuation and volumes change during abnormal periods is also an important topic in the stock market. As shown in Sect. 1, the price jumps suddenly with significant volume during the crisis periods, which reflects a crucial characteristic of stock market. In our simulation, this kind of relation can be straightforwardly observed. As illustrated in the Fig. 13, the dramatic changes of price (either positive or negative) are found with high trading volumes. When the price suddenly increases or decreases, the volumes almost double the daily average volume.

3.4.4 Informational Role of Trading Volume

As one of informational tools, trading volume is widely adopted by technicians. Investors identify specific signal from the volume to confirm the future price trend and discover the selling and buying opportunities. In efficient market, price movement is attributed to the available of new piece of information. The previous example of Google stock demonstrates the the informational role of trading volume to predict the future price trend. In the Fig. 14, we find similar price and volume pattern as in the real world. The trading volume of periods before $t = 436$ is less

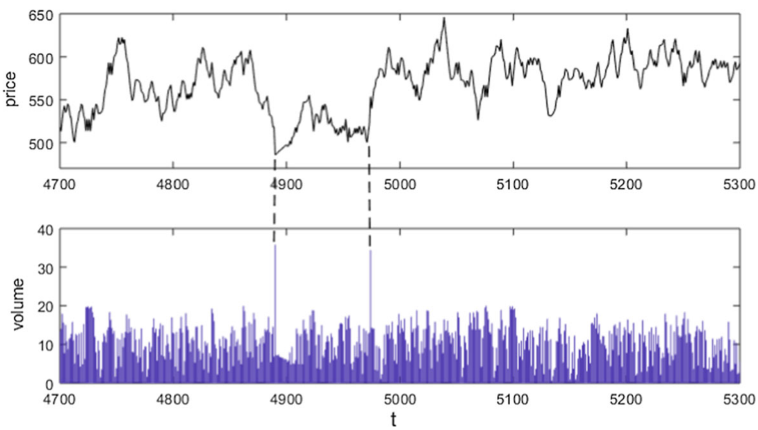

Fig. 13 Significant volume with large price change

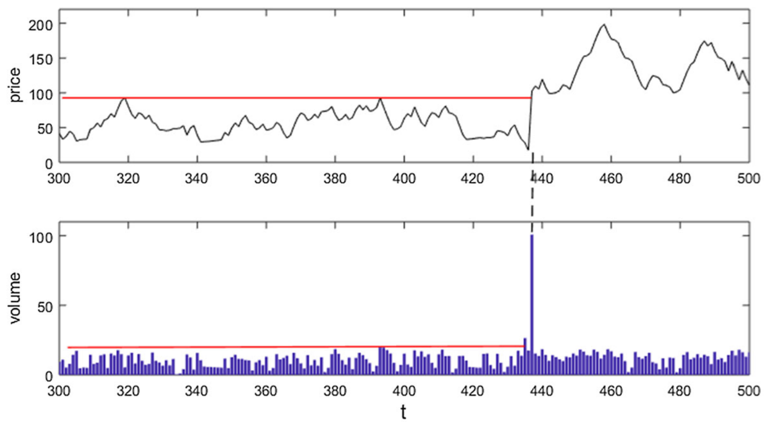

Fig. 14 Informational role of trading volume

than 20, and prices fluctuate in a window from 30 to 100. When the price suddenly jumps up to the next level at $t = 436$, the trading volume is almost five times the daily average, which confirms the breaking-out of price to the resistance line. After jumping up, prices move above the $p = 100$. This price level also becomes a new support line in technical analysis. The price jump with large volume can be explained by strong institutional buying power or speculative buying power as in Ülkü and Onishchenko (2019).

3.4.5 Robustness Check of Model's Fitness

To further investigate model's capability of generating "real" data, we try another entropy-related method to measure the statistical similarity between real financial data and simulated data. Kullback-Leibler Divergence (KL Divergence) is competent to measure the similarity of different variables. KL divergence has been popularly used in the data mining literature, and it was originated in probability theory and information theory. KL divergence is a non-symmetric measure of the difference between two probability distributions. If two variables are same, the value of KL divergence is zero. The smaller the value, the shorter the "distance" between two distributions. For more information on KL divergence, one can refer to Sankaran et al. (2016) and Kim and Sayama (2017). We adopt the method in the literature to calculate the KL divergence between S&P 500 data and simulated data for both return and trading volume, and the results are shown in Table 7. As presented in the table, divergences or distances between simulated data and "real"

Table 7 Kullback-Leibler divergence for return and volume

	Return	Volume
Simulate-real	0.561	0.132
Real-simulate	0.786	0.160
Mean of two directions	0.674	0.146

data are small for both return and volume, which provide the evidence of high similarity between stock market data and model simulated data. It also provides a new angle to measure the fitness of HAMs.

3.5 Asset Prices in Bubbles and Crises

Bubbles and crises have been the hot topic in financial market studies. As shown in overall price patterns in Fig. 6, our model can generate essential features of bubbles and crises, namely, large and growing overvaluation and undervaluation of the risky asset. Overvaluation (undervaluation) here means that the price exceeds (under) the fundamental value. To further explore the explanatory power of our model in bubbles and crises, we check whether large overvaluation and undervaluation hold for a wide range of parameter values. Figure 15 illustrates this. The four panels (a)–(d) correspond to four important parameters: adjustment speed of market maker γ; intensity of choice ρ; sensitiveness of fundamentalist a; and disagreements of chartists belief $\beta_1 - \beta_2$. In each panel, the upper diagram plots the maximum

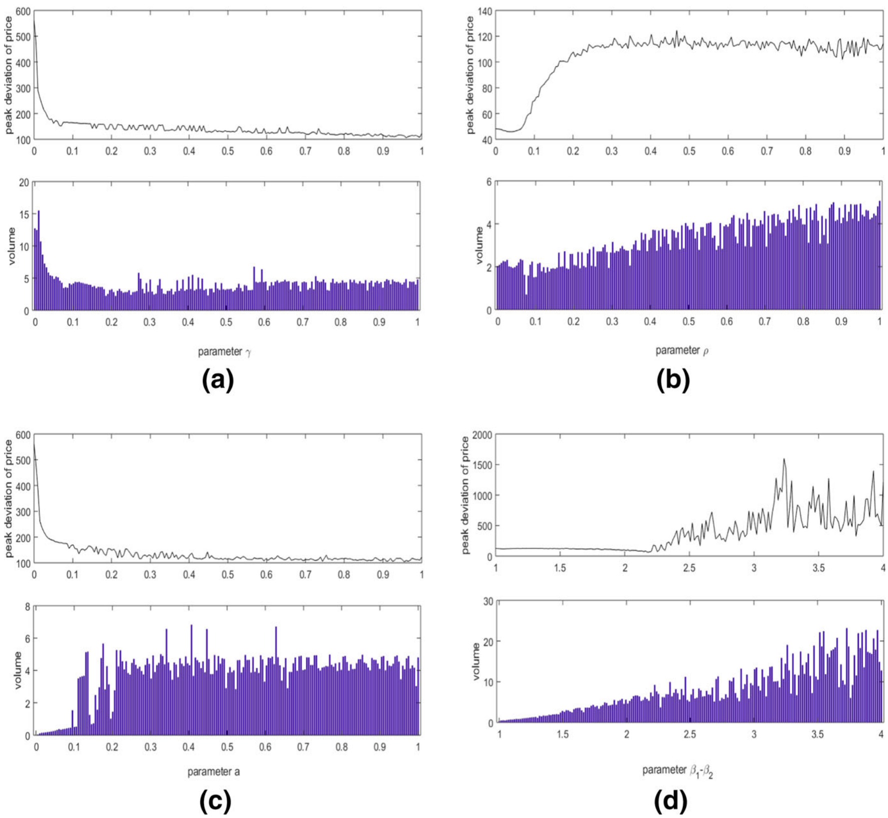

Fig. 15 The peak deviation of prices from fundamental values and corresponding volumes. **a** Variation of adjustment speed of market maker γ. **b** Variation of intensity of choice ρ. **c** Variation of sensitiveness of fundamentalist a. **d** Disagreements of chartists belief $\beta_1 - \beta_2$

deviation of the asset price, and the bottom diagram shows the corresponding trading volumes at the moment of peak deviation. For each panel, we generate maximum price deviations and volumes by varying the value of the parameter on the horizontal axis while keeping all other parameter values at the benchmark levels. The figure confirms that our model generates a large deviation from fundamental value for a wide range of parameter values. Panel (a) and (c) of the figure indicate that low values of γ and a increase the magnitude of deviation. In other words, bubbles or crises are more likely to occur with small price adjustment speed of market maker and low sensitiveness of fundamentalist. In financial markets, when the trend followers are less active and the fundamentalists are relatively more active, an increase in γ stabilizes the market price to the fundamental price, indicating the stabilizing role of the market maker. This finding is consistent with argument by Zhu et al. (2009) that when adjustment speed of market maker is small, he/she may act as a destabilizing force in the market. In addition, trading volumes keep the same trend with price deviation in panel (a) but present an opposite trend in panel (c). With the rise of sensitiveness of fundamentalists, the stabilizing role of fundamentalists will be amplified, and prices are more likely to be drew back to fundamental values, but the trading volume will be scaled up with the increasing demand from fundamentalists. In panel (b), increase of intensity of choice means that agents are able to quickly switch their trading strategies. With high clustering to one strategy of investors in the market, the possibility of bubbles and crises is very large. Most interestingly, the large disagreement of chartist's beliefs ($\beta_1 - \beta_2$) also generates large bubbles and crises with huge trading volumes.

3.6 Prices, Volumes and Beliefs Co-evolve in Different Chart Patterns and Crises

Besides simply looking at bubbles and crises, we are also interested in the co-evolvement of prices, volumes and investor beliefs in financial market. Some literature of HAM have investigated the co-movement of assets prices and agents beliefs, such as Boswijk et al. (2007) and Huang et al. (2010). Nevertheless, few of them have included the trading volumes into the analysis. In this section, we investigate the stock market from three dimensional viewpoints, and we observe the co-evolvement of prices, volumes and beliefs in our simulation. Furthermore, we introduce the chart patterns and crises into our analysis. As expected, our model has the ability to provide the reasonable explanation for the formation of different chart patterns and crises patterns.

Chart pattern is one of the most popular strategies for technicians, and they have summarized many patterns in the price series. Although not all of them appear during a specific period, some patterns such as double tops, double bottoms, head-and-shoulders and V tops are found in stock market frequently. We select some chart patterns from our simulation, and the co-evolvement of prices, volumes and beliefs within these pattern periods are displayed in Figs. 16 and 17. A double tops pattern is shown in the top of Fig. 16. At the first stage of the pattern, the fundamentalists are in charge of the market as the price is lower than their fundamental value. The great excess demand of fundamentalists pushes up the price,

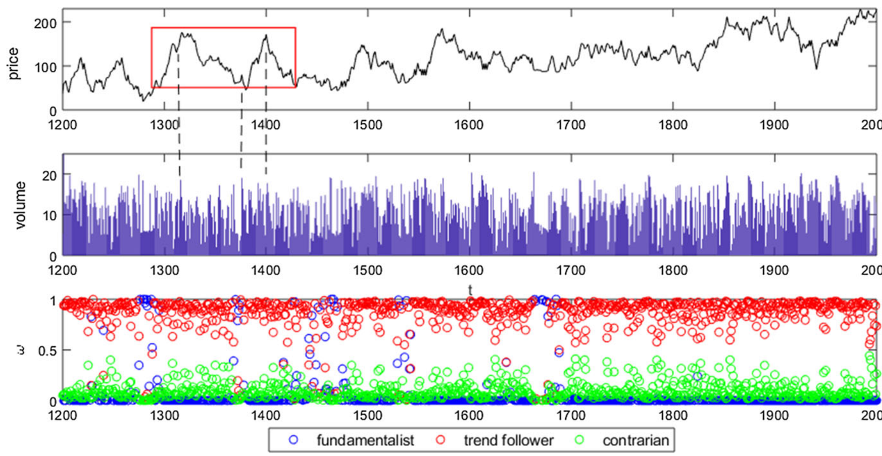

Fig. 16 Prices, volumes and beliefs co-evolve in double tops pattern

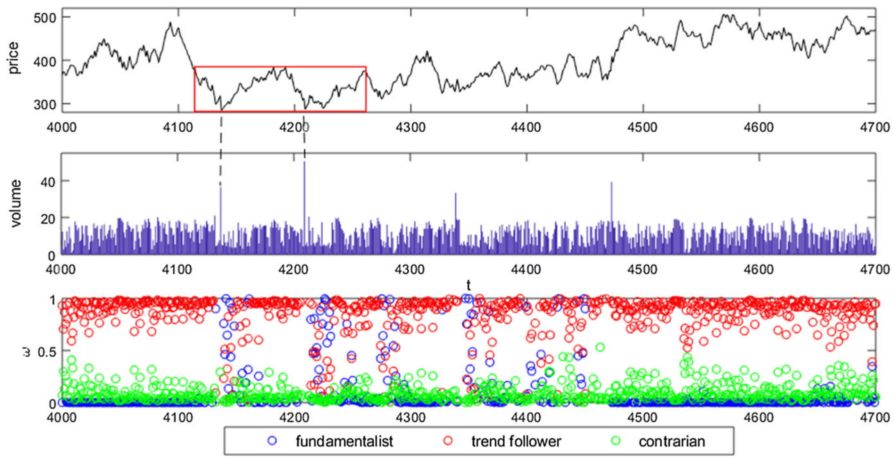

Fig. 17 Prices, volumes and beliefs co-evolve in double bottoms pattern

and when the price hikes gradually to the first top, the trend followers dominate the market. The gradual bubble occurs with large trading volume. At the next stage, when the price reaches a high level, the fraction of contrarians increases. It follows that price falls with more chartists chasing the falling trend. At the bottom, the price is lower than the fundamental value, and the fundamentalists take over the market again. Huge demand of fundamentalists leads to large trading volume, and the price rises again with the increasing number of chartist. Finally, the second top appears. Double bottoms pattern is reversal pattern of double tops. As shown in Fig. 17, the bottoms are formed with a sudden or smooth falling of price, and trading volume is also significant at the same time. The fundamentalists dominate the market at bottoms' period, and chartists take over during other periods.

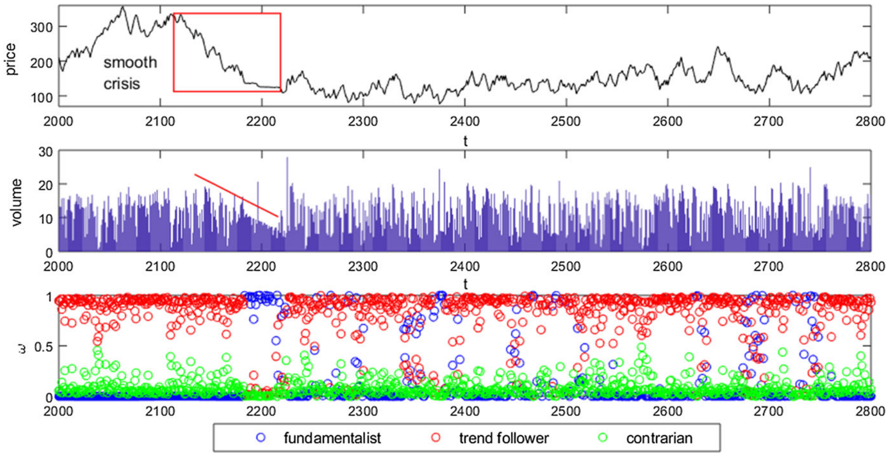

Fig. 18 Prices, volumes and beliefs co-evolve in a smooth crisis

HAMs have been widely used to explain the financial crises in many ways. Huang et al. (2010) have found that switches between trading strategies lead to price dynamic and cause different types of financial crises, such as sudden crisis, smooth crisis and disturbing crisis. We examine the smooth crisis and sudden crisis in our simulation, and the co-evolvement of prices, volumes and beliefs during crisis periods are demonstrated in Figs. 18 and 19. In the smooth crisis, the price falls smoothly from 300 to 100, and trading volume also decreases accordingly. When the price declines to a low level, fundamentalists replace the chartists to dominate the market. The lasting descending trading volume could be regarded as signal of smooth crisis. During smooth crisis, chartists dominate the market and induce the slow decline of price. The existence of contrarians slow down the speed of price

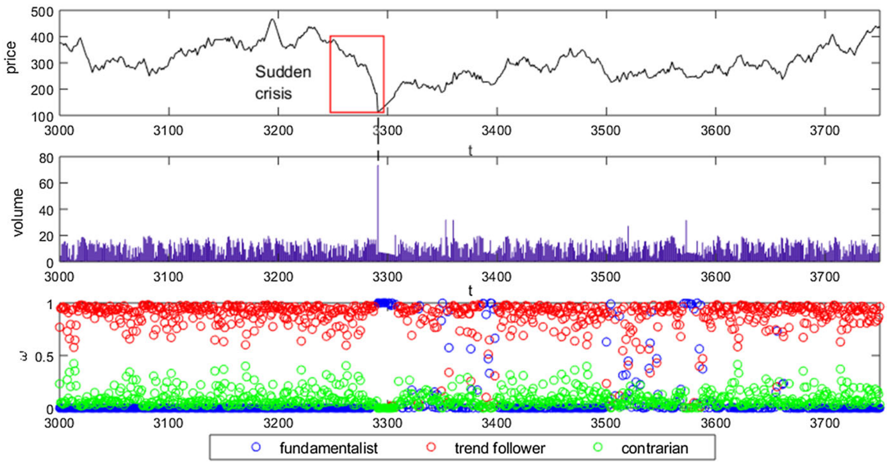

Fig. 19 Prices, volumes and beliefs co-evolve in a sudden crisis

falling. Once price falls below the fundamental value, fundamentalists begin to increase and take over the market. In a sudden crisis, the price plunges from the peak precipitately to the bottom, and trading volume surges at the same time. In the Fig. 19, the sudden crisis occurs with the spike shape volumes, and the number of fundamentalists suddenly increases and fundamentalists become the majority in the market.

4 Conclusion

In this paper, we develop a HAM with trading volume to replicate qualitative and quantitative features commonly observed in the stock market. Under the framework of market maker, fundamentalists and chartists hold heterogeneous beliefs on future price of risky assets. Agents are allowed to update their expected price based on different behaviors: the fundamentalists set their fundamental value referring to the costly internal information and economy growth rate, while chartists update their expected short-term fundamental value according to a series of psychological windows. To fit the real life case well, we introduce the adaptive evolutionary regime and agents freely switch to other group and choose the strategies that would optimize their discounted expected profit. The interaction between the fundamentalists and chartists could generate the price fluctuation and price-volume relationship. Meanwhile, the adaptive switching behavior of agents also increases market fluctuations both in price and volume. We are able to explore the rich dynamics of price and trading volume by building such a simple model, such as price bifurcations, V-shape price volume relation and features of bubbles and crises with variation of parameters.

Although we keep our model as simple as possible, it is also capable of generating a wide range of stylized facts both on price and volume simultaneously. As documented in literature, many HAMs are capable of generating stylized facts on price or return, such as unit root process in prices, fat tails, asymmetric and volatility clustering returns. The HAM in Huang and Zheng (2012) even has the ability to simulate the strict power-law distribution of return. After successfully simulating all the “standard” stylized facts above, we further explore the potential of our model in generating the stylized facts on volume. The deterministic model performs well in reproducing the stylized facts like stationary volume, positive correlation between volume and volatility, Granger causality between return and volume change. In addition, our model also closely mimics some visualized stylized facts which are commonly found in financial market, such as significant volume with dramatic price change and information role of trading volume. To demonstrate the power of our model in explaining different patterns in stock market, we identify different chart patterns and crises pattern that are frequently documented in technical analysis and literature, then analyze the formation of these patterns. We are the first who use tridimensional analysis approach to investigate the co-evolvement of prices, volumes and beliefs in these patterns. The co-evolvement of these three elements comprehensively reflects the trading activities and the investors’ behavior in financial market, which could give us a thorough view on financial market. By analyzing the chart patterns, our model could also provide theoretical underpinning for technical analysis.

Further exploration can be made as well. In the strategy of each group, none of them take the trading volume into their consideration. So the self-fulfilling power of volume signals could be test in the future study. In addition, the profitability of different trading strategies has been documented in many empirical literature, and we are interested to explore this feature by HAM in future research.

Appendix A

Linear Granger causality definition: Two stationary time series X_t and Y_t, let $F(X_t|\Omega_{t-1})$ be the conditional probability distribution of x_t given the bivariate information set Ω_{t-1} consisting of an L_X-length lagged vector of X_t and L_y-length lagged vector of Y_t. If:

$$F(X_t|\Omega_{t-1}) = F(X_t|(\Omega_{t-1} - Y^{l_y})), t = 1, 2, \cdots \tag{A.1}$$

Given lags L_x and L_y, the time series Y_t does not strictly Granger causality cause X_t. If the equality does not hold, Y is said to strictly Granger cause X. In plain words, X_t is said to Granger-cause Y_t if X cannot help predict future Y.

To test for Granger causality between stock return and volume change, we conduct the following vector autoregressive (VAR) model:

$$R_t = A + B(L)R_t + C(L)V_t + U_t \tag{A.2}$$

$$V_t = D + E(L)R_t + F(L)V_t + V_t \tag{A.3}$$

Where, R_t is stock return and V_t is percentage change of volume. B(L), C(L),E(L) and F(L) are lag polynomials of R_t and V_t.

Nonlinear Granger causality definition: Consider two strictly stationary and weakly dependent time series X_t, Y_t $t = 1, 2, 3\cdots$, We then denote the m-length lead vector of X_t by $\boldsymbol{X}_t^m$ and the Lx-length and Ly-length lag vectors of X_t and Y_t,respectively, by $\boldsymbol{X}_{t-Lx}^{Lx}$ and $\boldsymbol{Y}_{t-Ly}^{Ly}$.For given values of m, Lx, and $Ly \geq 1$ and for $e \geq 0$, Y does not strictly Granger-cause X if

$$\begin{aligned} &Pr(\| \boldsymbol{X}_t^m - \boldsymbol{X}_s^m \| \leq e \,\Big|\, \| \boldsymbol{X}_{t-Lx}^{Lx} - \boldsymbol{X}_{s-Lx}^{Lx} \| \leq e, \| \boldsymbol{Y}_{t-Ly}^{Ly} - \boldsymbol{Y}_{s-Ly}^{Ly} \| \leq e) \\ &\quad = Pr(\| \boldsymbol{X}_t^m - \boldsymbol{X}_s^m \| \leq e \,\Big|\, \| \boldsymbol{X}_{t-Lx}^{Lx} - \boldsymbol{X}_{s-Lx}^{Lx} \| \leq e) \end{aligned} \tag{A.4}$$

where $Pr(\cdot)$ denotes probability and $\| \cdot \|$ denotes the maximum norm. In order to transform equation (A.4) into a testable form, we denote the joint and marginal probabilities by:

$$
\begin{aligned}
C1(m+Lx, Ly, e) &\equiv Pr(\boldsymbol{X}_{t-Lx}^{m+Lx} - \boldsymbol{X}_{s-Lx}^{m+Lx} \parallel \leq e, \parallel \boldsymbol{Y}_{t-Ly}^{Ly} - \boldsymbol{Y}_{s-Ly}^{Ly} \parallel \leq e) \\
C2(Lx, Ly, e) &\equiv Pr(\boldsymbol{X}_{t-Lx}^{Lx} - \boldsymbol{X}_{s-Lx}^{Lx} \parallel \leq e, \parallel \boldsymbol{Y}_{t-Ly}^{Ly} - \boldsymbol{Y}_{s-Ly}^{Ly} \parallel \leq e) \\
C3(m+Lx, e) &\equiv Pr(\boldsymbol{X}_{t-Lx}^{m+Lx} - \boldsymbol{X}_{s-Lx}^{m+Lx} \parallel \leq e) \\
C4(Lx, e) &\equiv Pr(\boldsymbol{X}_{t-Lx}^{Lx} - \boldsymbol{X}_{s-Lx}^{Lx} \parallel \leq e)
\end{aligned}
$$

The strict Granger noncausality condition in equation (4) can be expressed as

$$
\frac{C1(m+Lx, Ly, e)}{C2(Lx, Ly, e)} = \frac{C3(m+Lx, e)}{C4(Lx, e)} \tag{A.5}
$$

The null hypothesis for Y_t strictly Granger-causing X_t in Eq. (A5) is

Table 8 Linear Granger causality test

Dependent variable	S&P 500 (1) R_t	S&P 500 (2) V_t	Dow Jones Index (3) R_t	Dow Jones Index (4) V_t
Constant	−0.660	5.321***	−1.301	3.168***
	(2.664)	(0.478)	(1.143)	(0.361)
R_{t-1}	−0.047*	−0.011***	−0.048**	−0.026***
	(0.024)	(0.004)	(0.024)	(0.008)
R_{t-2}	0.020	-0.003	0.031	−0.016**
	(0.024)	(0.004)	(0.024)	(0.008)
R_{t-3}	-0.066***	−0.002	−0.042*	-0.006
	(0.024)	0.004	(0.024)	(0.008)
R_{t-4}	−0.005	−0.010	−0.015	-0.004
	(0.024)	(0.004)	(0.024)	(0.008)
V_{t-1}	−0.104	0.471***	−0.011	0.378***
	(0.133)	(0.024)	(0.075)	(0.024)
V_{t-2}	0.252*	0.456***	0.134*	0.188***
	(0.147)	(0.026)	(0.080)	(0.025)
V_{t-3}	−0.247*	0.051*	−0.084	0.152***
	(0.147)	(0.026)	(0.081)	(0.025)
V_{t-4}	0.131	0.079***	0.032	0.113***
	(0.133)	(0.024)	(0.076)	(0.024)
Wald-stat	7.923	11.340***	4.218	15.678***
p-value	0.295	0.023	0.377	0.003

Notes: (1) The entries in brackets are the standard errors. The Wald-stat and p-value are tests of Granger causality. (2)* denotes rejection at the 10% level,** rejection at the 5% level, ***rejection at the 1% level

Table 9 Nonlinear Granger causality test

Lags	H_0:Stock Returns Do not Cause Volume Changes		H_0:Volume Changes Do not Cause Stock Returns	
Lx=Ly S&P 500	T-statistic	p-value	T-statistics	p-value
1	3.027	0.001	1.059	0.145
2	2.237	0.013	0.960	0.169
3	1.975	0.024	0.403	0.343
4	2.076	0.019	1.241	0.101
5	1.175	0.120	1.115	0.132
6	0.748	0.227	1.372	0.085
7	0.966	0.167	1.924	0.027
8	0.566	0.286	1.575	0.058
Dow Jones Index				
1	1.026	0.153	−0.613	0.730
2	0.874	0.191	−0.575	0.717
3	0.128	0.449	0.248	0.402
4	0.249	0.401	−0.041	0.516
5	0.609	0.271	−0.086	0.534
6	−0.436	0.668	0.118	0.453
7	−0.422	0.663	0.239	0.405
8	−0.222	0.588	0.872	0.192

Note:Test critical value for T statistics are 2.326 (1%),1.645 (5%) and 1.282 (10%)

$$\sqrt{n}\left(\frac{C1(m+Lx,Ly,e,n)}{C2(Lx,Ly,e,n)}-\frac{C3(m+Lx,e,n)}{C4(Lx,e,n)}\right) \sim N(0,\sigma^2(m,Lx,Ly,e)) \tag{A.6}$$

After getting two estimated residual series U_t and V_t from the linear VAR estimation, we use the modified HJ test(see Diks and Panchenko (2006)) to investigate the nonlinear Granger causality between stock return and trading volume. The results of linear and nonlinear Granger causality tests are shown in Tables 8 and 9, respectively.

Appendix B

The chance function shows the chance of lost opportunity either to buy when the assets price is low or fail to sell when the price is high. It can be expressed as

$$A(\mu_t^f, p_t) = a(p_t - m(\mu_t^f))^d (M(\mu_t^f) - p_t)^d \tag{B.1}$$

where a and d are the parameters that describe the sensitiveness of fundamentalists

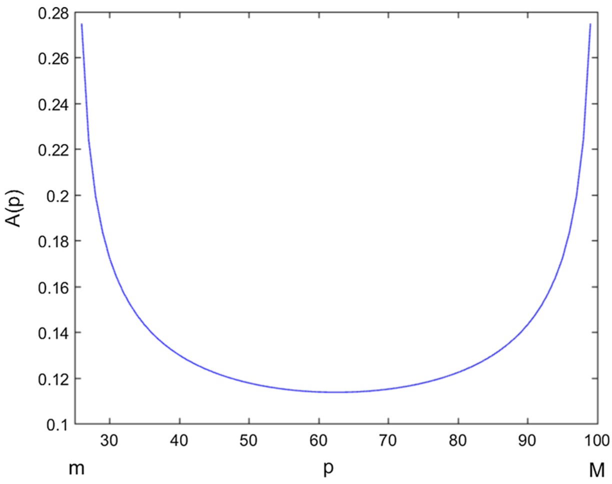

Fig. 20 The chance function

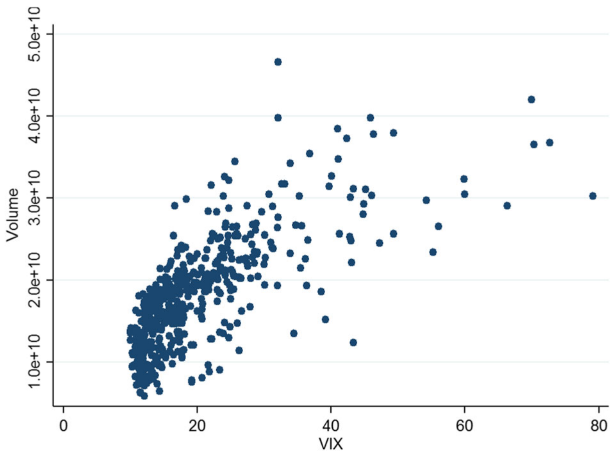

Fig. 21 Correlation between volume and the VIX

when the price move close to the boundaries and $a > 0$, $d < 0$. Assuming μ_t^f as constant, the chance function can be simply illustrated as Fig. 20.

We define the boundaries as below

$$M_t = k\mu_t^f \text{ and } m_t = \frac{1}{k}\mu_t^f \tag{B.2}$$

As defined in Black (1986), price fluctuates within a reasonable bond in efficient market. $k > 1$ is a pre-selected factor and μ_t^f is the fundamental value of risky assets.

Appendix C

See Fig. 21

Scatter plotting of S&P 500 trading volume and VIX between 1/1/2010 and 12/31/2016.

Appendix D

In Fig. 22a, the sensitivity of fundamentalists when price moves close to the boundaries (a) is used as a bifurcation parameter, the price dynamics are stable for $a \leq 1.3$. However, when a goes beyond this value, the equilibrium begins to lose its stability with a stable period-2 cycle. When a increases further, chaos takes place through a cascade of infinite sequence of period-doubling bifurcation. When

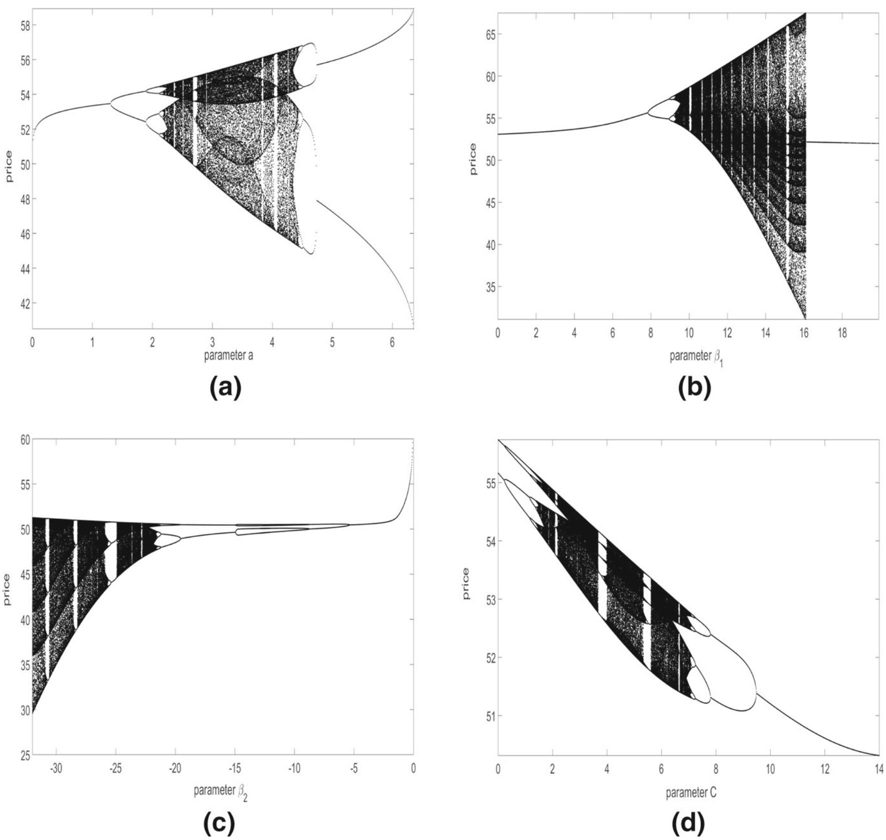

Fig. 22 Dynamics of the model. **a** Bifurcation diagram for sensitiveness of fundamentalist. **b** Bifurcation diagram for sensitiveness of trend follower. **c** Bifurcation diagram for sensitiveness of contrarian. **d** Bifurcation diagram for information cost of fundamentalist

a approaches 4.8, the price dynamics become stable again and a reversed flip bifurcation is observed.

Figure 22b shows a similar periodic bifurcation when the sensitivity of the price expectation of trend followers to past estimation bias (β_1) is used. The price dynamics are stable when the value of β_1 is small and cascade of flip bifurcation occurs before it leads to chaotic motion. The dynamics will eventually stabilize when β_2 goes beyond 16. Figure 22c is the bifurcation diagram when the sensitivity of the price expectation of contrarians to past estimation bias (β_2) is used as the parameter of the analysis. As β_2 holds on to a negative value, β_2 decreases from 0 means that the sensitivity of contrarians to past estimation bias increases. Again we see that when contrarians are more sensitive to past estimation bias, i.e. β_2 is more negative, a period-2 cycle first emerges followed by a period-4 cycle before it generates chaos.

Information cost may affect the model dynamics through the expected profit function of fundamentalist. As shown in panel (d), if information cost C is small then there exists a stable period-2 cycle. If C increases, this cycle becomes unstable and flip bifurcations occur. After infinitely many flip bifurcations the price behaviour becomes chaotic, as C increases. When C further increases after a cascade of period halving bifurcations there exists stable stationary equilibrium.

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